Name:__________________________________________________________

Score: ___________ / 250 points

Percentage: ___________ / 100 %

• Show all logical work and clearly label graphs to receive full credit.

• Draw a box around your final answer(s).

• Give exact answers, unless stated otherwise.

• Where appropriate, round correctly to 2 decimal places.

• Be sure to check your solutions.

• The back of this page contains blank grids for scratch work.

• Two pages follow for scratch work.
Blank Grids for Scratch Work
Scratch Work Page
1. [8 pt.] Solve the equation by factoring. Find all real solutions.
\[ 6x^3 + 27x^2 - 15x = 0 \]

2. Given \( \frac{1}{x-4} - \frac{3}{x} = \frac{4}{x(x-4)} \)
   
   a. [2 pt.] State the domain.
   
   b. [6 pt.] Solve the equation. Find all real solutions. (Be sure to check for extraneous solutions.)

3. [8 pt.] Solve the equation. Find all real solutions. (Be sure to check for extraneous solutions.)
\[ x = 1 + \sqrt{7 - x} \]

4. [8 pt.] Solve the equation by using the quadratic formula. Find all real and complex solutions. Simplify completely.
\[ x^2 = 6x - 21 \]
5. [8 pt.] Solve the following inequality. Write the solution in interval notation.

\[ 2|3x + 5| - 4 > 10 \]

6. [8 pt.] Solve the system of equations. Write your solution as an ordered pair.

\[ \begin{align*}
3x + y &= 5 \\
-2x + 3y &= -18
\end{align*} \]

7. [8 pt.] Solve the equation for \( x \). Find all real solutions.

\[ 2^{7x - 24} = \frac{1}{8} \]

8. [10 pt.] Use properties of logarithms to solve the equation for \( x \). (Be sure to check for extraneous solutions.)

\[ \log_6 (x^2 + 5x) - 2\log_6(6) = 0 \]
9. Given \( f(x) = \frac{1}{x} \) and \( g(x) = \sqrt{x + 6} \)
   
   a. [4 pt.] Find the domain of \( g(x) \).

   b. [4 pt.] Find the following. Write your answer as a single fraction.
   \[
   (f + g)(x) =
   \]

   c. [4 pt.] Evaluate the following. Write your answer as fraction in lowest terms.
   \[
   (f + g)(-5) =
   \]

   d. [4 pt.] Find the following. Write your answer as a single fraction.
   \[
   f(g(x)) =
   \]

   e. [4 pt.] Evaluate the following. Write your answer as fraction in lowest terms.
   \[
   f(g(10)) =
   \]

10. a. [2 pt.] Find the slope of the linear function that goes through the points \((0, 4)\) and \((6, -1)\).

   b. [4 pt.] Find the equation of the linear function that goes through the points \((0, 4)\) and \((6, -1)\). Write your answer in slope-intercept form.

   c. [4 pt.] Find the equation of the linear function that is perpendicular to the line you found in part (b) and passes through the point \((2, -3)\). Write your answer in slope-intercept form.
11. Start with the graph of $f(x)$ which is pictured below.

a. [4 pt.] List the transformations necessary to obtain the function $g(x)$.

$$g(x) = -\frac{1}{2}f(x + 2) - 3$$

b. [3 pt.] The key points on the graph of $f(x)$ are given below. What are these points transformed to on the graph of $g(x)$?

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$g(x) = -\frac{1}{2}f(x + 2) - 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>($-3$, 0)</td>
<td>$-\frac{1}{2}f(-1) - 3$</td>
</tr>
<tr>
<td>($-2$, 4)</td>
<td>$-\frac{1}{2}f(0) - 3$</td>
</tr>
<tr>
<td>($3$, 2)</td>
<td>$-\frac{1}{2}f(5) - 3$</td>
</tr>
</tbody>
</table>

c. [4 pt.] Graph the transformed function $g(x)$ on the same axes.

12. The equation and graph of a piece-wise function $f(x)$ is given below.

$$f(x) = \begin{cases} 
|x + 3| & \text{if } -6 \leq x \leq -1 \\
 x^2 - 1 & \text{if } x > -1 
\end{cases}$$

a. [4 pt.] Find the domain of $f(x)$. Use interval notation.

b. [2 pt.] Find the range of $f(x)$. Use interval notation.

c. [4 pt.] Find the $x$-intercept(s) of $f(x)$. Write your answer(s) as points.

d. [2 pt.] Find the $y$-intercept of $f(x)$. Write your answer as a point.

e. [3 pt.] Evaluate the following, if possible. If not, write DNE.

$$f(-7) = \quad f(-1) = \quad f(3) =$$
13. Given \( f(x) = 0.4x^4 - 4.5x^2 + 8.1 \)

*For this entire question, round to 2 decimal places when necessary.*

a. [2 pt.] Use a graphing calculator to sketch the graph of \( f(x) \).

b. [4 pt.] Find the \( x \)-intercept(s) of \( f(x) \).
   Write your answer(s) as points.

c. [2 pt.] Find the \( y \)-intercept of \( f(x) \).
   Write your answer as a point.

d. [4 pt.] Is \( f(x) \) even, odd, or neither? Explain your reasoning.

e. [2 pt.] The graph has a local (relative) minimum of _________ at \( x = \) ________.

f. [2 pt.] The graph has a local (relative) minimum of _________ at \( x = \) ________.

g. [2 pt.] The graph has a local (relative) maximum of _________ at \( x = \) ________.

h. [4 pt.] On what interval(s) is \( f(x) \) increasing?
14. Consider the function \( f(x) = -7x^4(x - 3)(x + 6)^5. \)

a. [4 pt.] Find the domain of \( f(x). \)

b. [2 pt.] State the degree of \( f(x). \)

c. [2 pt.] State the leading term of \( f(x). \)

d. [2 pt.] Sketch the end behavior of the graph of \( f(x). \)

![End Behavior Diagram]

e. [8 pt.] Sketch a possible graph of \( f(x). \) Clearly label the zeros and show if the graph touches or crosses the \( x \)-axis at each zero.

<table>
<thead>
<tr>
<th>Zeros</th>
<th>Multiplicity</th>
<th>Behavior (Touch or Cross?)</th>
</tr>
</thead>
</table>

![Graph Sketch]

15. Consider the function \( f(x) = \frac{2x^2 + 2x - 12}{x^2 - 4} \).

Notice that the factored form of the function is

\[
\begin{align*}
f(x) &= \frac{2(x + 3)(x - 2)}{(x + 2)(x - 2)}
\end{align*}
\]

a. [4 pt.] Find the domain of \( f(x) \).

b. [2 pt.] Find the horizontal asymptote of \( f(x) \).

c. [2 pt.] Find the vertical asymptote(s) of \( f(x) \).

d. [2 pt.] Find the coordinate point for any hole(s) of \( f(x) \).

e. [2 pt.] Find the \( x \)-intercept(s), if any, of \( f(x) \).

f. [2 pt.] Find the \( y \)-intercept, if any, of \( f(x) \).

g. [6 pt.] Sketch the graph of \( f(x) \). Clearly label the intercepts, asymptotes, and holes.
16. [8 pt.] Solve the following inequality. Show your work by using an appropriate number line with test points or by sketching the graph. Write the solution in interval notation.

\[
\frac{x-6}{x-3} \leq 0
\]

17. The graph \( f(x) = e^x \) is given below.

a. [6 pt.] Sketch \( f^{-1}(x) \) on the same axes and label three key points on the graph.

b. [2 pt.] What is the range of \( f^{-1}(x) \)? Use interval notation.

18. Given the function \( f(x) = \frac{2x+1}{x-5} \)

a. [8 pt.] Find the inverse function \( f^{-1}(x) \).

b. [2 pt.] State the equation of the function \( f^{-1}(x) \).

\[ f^{-1}(x) = \]
19. [8 pt.] When a softball player hits a ball, the height of the ball depends upon the time it has been in flight. Alicia hits a softball modeled by the equation

\[ h(t) = -16t^2 + 96t + 6 \]

where \( h \) is feet above the ground and \( t \) is time in seconds.

What is the maximum height reached by the softball? Solve algebraically.

21. Carson takes 500 mg of acetaminophen. The amount of acetaminophen in her body can be modeled by a continuous exponential decay function, with an hourly decay rate of 24%.

a. [4 pt.] Find the exponential decay function.

b. [6 pt.] Determine how long it will take for there to be only 1 mg of acetaminophen in Carson’s system. (Round to two decimal places.)

20. [8 pt.] Julian just inherited $50,000 from his grandmother. He is going to invest all of it in a new savings account at 3%, which is compounded monthly.

If Julian does not make any withdrawals or deposits, how much money will he have in this savings account in 23 years? (Round to two decimal places.)
22. [8 pt.] Find the difference quotient for $f(x) = 3x^2 - 2x$.

(Part C is required. Parts A and B are optional, but may be helpful for Part C.)

A. $f(x + h) =$

B. $f(x + h) - f(x) =$

C. $\frac{f(x + h) - f(x)}{h} =$


23. [8 pt.] The owner of a candy store makes a new taffy, called Seahawk Taffy, by mixing lemon taffy that costs $1.50 per pound and lime taffy that costs $1.70 per pound. How many pounds of each flavor of taffy should the owner mix to obtain 100 pounds of Seahawk Taffy that will cost $1.62 per pound without changing revenue? For full credit, you must show algebraic work to support your solution.
Thanks for a great semester! 😊