General test instructions: Show all your work on this test paper! If you solve a problem algebraically show all your steps. If you solve a problem by graphing on your calculator, show a sketch of the graph, with the solution labeled. Where appropriate, round answers to 3 decimal places.

1. Solve: \( x^2 + 8x = 4 \)
2. Solve: \( x = \sqrt{15 - 2x} \)

3. Solve this system of equations.
   \[
   \begin{align*}
   9x + 3y &= 1 \\
   3x - 6y &= 5
   \end{align*}
   \]

4. Solve this system of equations.
   \[
   \begin{align*}
   x^2 &= y - 1 \\
   y &= 3x + 5
   \end{align*}
   \]

5. One day a chemist needed a 20% solution of potassium permanganate. She had a 15% solution on hand as well as a 30% solution. How many liters of the 15% solution should she add to 3 liters of the 30% solution to get her 20% solution?

6. Solve: \( |2 - 7m| - 1 < 4 \)

7. Solve: \( x^2 - x - 12 \geq 0 \)

8. Evaluate:
   a. \( 5 \ln e^3 = \)
   b. \( \log_4 25 = \)

9. Solve: \( 5e^{-x} + 5 = 23 \)

10. Solve: \( \log (x) + \log (x+15) = 2 \)

11. John invested $600 at 5.5% compounded continuously.
   a. How long would it take John to triple his money?
   b. How much money will John have at the end of 10 years?

12. a. Graph \( f(x) = \log_2 x \) and \( g(x) = \log_2 (x-1) + 3 \) (Label intercepts and asymptotes.)
   b. Describe how the graph of \( g(x) \) differs from that of \( f(x) \) in terms of transformations such as shifts up or down, left or right.

13. Write an equation of a third degree polynomial with a negative leading coefficient and zeros \(-2, 3, \) and \(1.\)

14. List all real zeros (including the multiplicity for each) for the following polynomials:
   a. \( f(x) = x^3 - 3x + 2 \)
   b. \( f(x) = 2x (2x + 3)(x^2 - 1) \)

15. A company has found that the annual revenue from sales of cell phones is a function of the unit price, \( p, \) that it charges. If the revenue, \( R, \) is \( R(p) = -150p^2 + 12,000p \)
   a. What price should be charged to maximize revenue?
   b. What is the maximum revenue?

16. Analyze the function \( f(x) = \frac{2x^2}{x^2 - 4} \)
   a. \( y \)-intercept(s)
   b. \( x \)-intercept(s)
   c. vertical asymptote(s)
   d. horizontal asymptote(s)
   e. graph \( y = f(x); \) include \( x \) and \( y \)-intercepts and all asymptotes.
17. Simplify; write in the standard \( a + bi \) form.
\[
\frac{2 + 4i}{1 - 2i}
\]

18. Find ALL real and complex zeros of the polynomial function
\[
F(x) = (x^3 - 1) (x^2 + 4x + 5)
\]

19. The graph of \( y = f(x) \) is shown. On the same set of axes, sketch the graph of
\[
y = -f(x + 2).
\]

20. Given the function \( g(x) = \sqrt{x - 2} \), what is its
   a. domain?
   b. range?
   c. What is the inverse function of \( g(x) \)?
   d. Graph \( g(x) \) and \( g^{-1}(x) \).

21. Given
\[
f(x) = 5 - x \quad \text{and} \quad g(x) = x^2 + 4
\]
find:
   a. \( f(-11) \)
   b. \( (f - g)(-4) \)
   c. \( (g \circ f)(x) \).

22. Find an equation of the line that passes through the point \((2, -1)\) and is perpendicular to the line \(6x - 3y = 15\).

23. The following data represent the population of a colony of bacteria during a 10 day period.

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1250</td>
</tr>
<tr>
<td>2</td>
<td>840</td>
</tr>
<tr>
<td>4</td>
<td>560</td>
</tr>
<tr>
<td>6</td>
<td>380</td>
</tr>
<tr>
<td>8</td>
<td>250</td>
</tr>
<tr>
<td>10</td>
<td>170</td>
</tr>
</tbody>
</table>

   a. Using a graphing utility, draw a scatter diagram.
   b. Find the line of best fit to the data.
      Write the linear function here:
   c. Fit an exponential curve to the data.
      Write the exponential function here:
   d. On the same axes as your scatter-plot in part a, graph the best fit line and the best fit exponential curve. Which of the two models fits the data better?
   e. Using the model you chose in part (d), estimate the population of the colony at 12 days.

24. Given the four functions \( f, g, h \) and \( k \):
\[
f(x) = |x + 1|, \quad g(x) = -(x^2 + 3), \quad h(x) = \sqrt{x+1}, \quad \text{and} \quad k(x) = \frac{3x}{x^2 + 1}
\]
   a. Which function or functions have an inverse function? (That is, which function(s) are one-to-one?)
   b. Which function or functions are even? (That is, which are symmetric with respect to the y-axis?)
   c. Which function or functions are increasing on the interval \((-1, \infty)\)?

25. a. Graph the function
\[
F(x) = \begin{cases} 
  x^2, & \text{if } x \leq 1 \\
  2, & \text{if } x > 1
\end{cases}
\]
   b. Evaluate \( f(0) \)
   c. Evaluate \( f(3) \)