From Here to Eternity and Back: Are Traversable Wormholes Possible?

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with advising from
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Dedicated in Memory of My Dear Friend and UNCW Alumnus
Do Physicists Care, and Why?

- Are wormholes possible?
  - How can we model to prove or disprove?
  - Wormholes are being taken seriously

- Possible uses?
  - Interstellar travel
  - Time travel
Outline

- History
- Models that fail
- Desirable traits
- General relativity primer
- Morris-Thorne wormhole
  - Curvature
  - Stress energy tensor
  - Boundary conditions and embeddings
  - Geodesics
  - Examples
- Time machines and future research
What is a Wormhole?

- “Hypothetical shortcut between distant points”
- General relativity
Historical Perspective

- Albert Einstein – general relativity ~ 1916
- Karl Schwarzschild – first exact solution to field equations ~ 1916
  - Unique spherically symmetric vacuum solution
- Ludwig Flamm ~ 1916
  - White hole solution
Historical Perspective

- Einstein and Nathan Rosen ~ 1935
  - “Einstein-Rosen Bridge” – first mathematical proof

- Kurt Gödel ~ 1948
  - Time tunnels possible?

- John Archibald Wheeler ~ 1950’s
  - Coined term, “wormhole”

- Michael Morris and Kip Thorne ~ 1988
  - “Most promising” – wormholes as tools to teach general relativity
Traversable Wormholes?

- Matter travels from one mouth to other through throat
- Never observed
- BUT proven valid solution to field equations of general relativity
Black Holes Not the Answer

- Tidal forces too strong
- Horizons
  - One-way membranes
  - Time slows to stop
- "Schwarzschild wormholes"
  - Fail for same reasons
Simple Models – Geometric Traits

- Everywhere obeys Einstein’s field equations
- Spherically symmetric, static metric
- “Throat” connecting asymptotically flat spacetime regions
- No event horizon
  - Two-way travel
  - Finite crossing time
Simple Models – Physical Traits

- Small tidal forces
- Reasonable crossing time
- Reasonable stress-energy tensor
- Stable
- Assembly possible
A Little General Relativity Primer

\[ G_{\alpha\beta} = 8\pi GT_{\alpha\beta} \]

\[ G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} Rg_{\alpha\beta} \]

- Field equations – relate spacetime curvature to matter and energy distribution
- Left side – Curvature
- Right side – Stress energy tensor
- Summation indices
More on General Relativity

“Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve.” ~ Misner, Thorne, Wheeler, *Gravitation*
Spacetimes

“A spacetime is a four-dimensional manifold equipped with a Lorentzian metric...[of signature]...(-,+,+,+). A spacetime is often referred to as having (3+1) dimensions.” ~Matt Visser, *Lorentzian Wormholes*. 
Representing Spacetimes: Flat

“Line Element”

- Uses differentials
- Einstein summation
- Metric
- Minkowski space:

\[ ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \]

\[ ds^2 = g_{tt} dt^2 + g_{xx} dx^2 + g_{yy} dy^2 + g_{zz} dz^2 \]

\[ g_{tt} = -1; \quad g_{xx} = 1; \quad g_{yy} = 1; \quad g_{zz} = 1 \quad (\text{cartesian}) \]

\[ g_{tt} = -1; \quad g_{rr} = 1; \quad g_{\theta\theta} = r^2; \quad g_{\phi\phi} = r^2 \sin^2 \theta \quad (\text{spherical}) \]
Morris-Thorne Wormhole (1988)

\[ ds^2 = -e^{2\Phi} dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

- \( \Phi(r) \) – redshift function
  - Change in frequency of electromagnetic radiation in gravitational field

- \( b(r) \) – shape function
Properties of the Metric

\[ ds^2 = -e^{2\Phi} dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)^2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

- Spherically symmetric and static
- Radial coordinate \( r \) such that circumference of circle centered around throat given by \( 2\pi r \)
- \( r \) decreases from \( +\infty \) to \( b=b_0 \) (minimum radius) at throat, then increases from \( b_0 \) to \( +\infty \)
- At throat exists coordinate singularity where \( r \) component diverges
- Proper radial distance \( l(r) \) runs from \( -\infty \) to \( +\infty \) and vice versa
Morris-Thorne Metric

\[ ds^2 = -e^{2\Phi} dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[
g_{\alpha\beta} = \begin{bmatrix} g_{tt} & 0 & 0 & 0 & 0 \\ 0 & g_{rr} & 0 & 0 & 0 \\ 0 & 0 & g_{\theta\theta} & 0 & 0 \\ 0 & 0 & 0 & g_{\phi\phi} & 0 \end{bmatrix} = \begin{bmatrix} -e^{2\Phi} & 0 & 0 & 0 & 0 \\ 0 & \left(1 - \frac{b}{r}\right)^{-1} & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta & 0 \end{bmatrix} \]
Determining Curvature (Left Side)

- Do we have the desired shape?
- Cartan’s Structure Equations

\[ G_{\alpha\beta} = 8\pi GT_{\alpha\beta} \]

“Cartan I”

\[ d\Theta^i = -\omega^i_j \wedge \Theta^j \]

“Cartan II”

\[ \Omega^i_j = d\omega^i_j + \omega^i_k \wedge \omega^k_j \]

Elie Joseph Cartan (1869~1951)
“One-forms”

\[ dx, dy, dz \]

\[ pdx + qdy + rdz \]

\[ \omega^i_j = -\omega^j_i \]

Take from Morris-Thorne metric:

\[ \Theta^0 = -e^\Phi dt \]

\[ \Theta^1 = \left(1 - \frac{b}{r}\right)^{-\frac{1}{2}} dr \]

\[ \Theta^2 = rd\theta \]

\[ \Theta^3 = r\sin\theta d\phi \]
Operations with Forms

- "Wedge product"
  \[ dt \wedge dt = dr \wedge dr = d\theta \wedge d\theta = d\phi \wedge d\phi = 0 \]
  \[ dt \wedge dr = -dr \wedge dt \]

- "d" Operator: \(k\)-form to \((k+1)\)-form
  1-form
  \[ a = a(t,r) \Rightarrow da = \frac{da}{dt} dt + \frac{da}{dr} dr \]

- Combining:
  \[ \omega = a(t,r) dt \Rightarrow d\omega = \frac{da}{dt} dt \wedge dt + \frac{da}{dr} dr \wedge dt = \frac{da}{dr} dr \wedge dt \]
Calculation – Connection One-Forms

\[ ds^2 = -e^{2\Phi} dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)^{\frac{1}{2}}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ \Theta^0 = -e^\Phi dt \Rightarrow d\Theta^0 = \Phi' e^\Phi dt \wedge dr; \quad dt = \frac{1}{e^\Phi} \Theta^0 \]

\[ \Theta^1 = \left(1 - \frac{b}{r}\right)^{-\frac{1}{2}} dr \Rightarrow d\Theta^1 = 0; \quad dr = \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} \Theta^1 \]

\[ \Theta^2 = rd\theta \Rightarrow d\Theta^2 = dr \wedge d\theta; \quad d\theta = \frac{1}{r} \Theta^2 \]

\[ \Theta^3 = r \sin \theta d\phi \Rightarrow d\Theta^3 = \sin \theta dr \wedge d\phi + r \cos \theta d\theta \wedge d\phi; \quad d\phi = \frac{1}{r \sin \theta} \Theta^3 \]
Calculation – Connection One-Forms

\[ d\Theta^i = -\omega_j^i \wedge \Theta^j \]

\[ i, j = t, r, \theta, \phi \]

\[ d\Theta^0 = -\omega_1^0 \wedge \Theta^1 - \omega_2^0 \wedge \Theta^2 - \omega_3^0 \wedge \Theta^3 = \Phi' e^\Phi dt \wedge dr \]

\[ \Rightarrow -\omega_1^0 \wedge \Theta^1 = \Phi' e^\Phi dt \wedge dr \Rightarrow \omega_1^0 \wedge \Theta^1 = -\Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt \wedge \Theta^1 \]

\[ \Rightarrow \omega_1^0 = -\Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt \]

\[ \Rightarrow \omega_0^1 = \Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt \]
Matrix of One-Forms

\[ \omega_j^i = \begin{bmatrix}
0 & -\Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt & 0 & 0 \\
\Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt & 0 & -\left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\theta & -\sin \theta \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\phi \\
0 & \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\theta & 0 & -\cos \theta d\phi \\
0 & \sin \theta \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\phi & \cos \theta d\phi & 0
\end{bmatrix} \]
Calculation – Curvature Two-Forms

\[ \Omega^i_j = d\omega^i_j + \omega^i_k \wedge \omega^k_j \]

\[ = -\frac{1}{2} R^i_{mnj} \Theta^m \wedge \Theta^n \]

- Computed using matrix of one-forms
- Non-zero components of Riemann tensor
- Useful for computing geodesics
Calculation – Curvature Two-Forms

\[
\omega_3^2 = -\cos \theta d\phi \\
d\omega_3^2 = \sin \theta d\theta \wedge d\phi \\
\omega_0^2 = \omega_3^0 = 0 \\
\omega_1^2 = \left(1 - \frac{b}{r}\right)^2 d\theta \\
\omega_3^1 = -\sin \theta \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\phi \\
d\theta = \frac{1}{r} \Theta^2 \\
d\phi = \frac{1}{r \sin \theta} \Theta^3
\]

\[
\Omega_3^2 = d\omega_3^2 + \omega_0^2 \wedge \omega_3^0 + \omega_1^2 \wedge \omega_3^1 \\
= \frac{b}{r} \sin \theta d\theta \wedge d\phi \\
= \left(\frac{b}{r} \sin \theta\right) \left(\frac{1}{r}\right) \left(\frac{1}{r \sin \theta}\right) \Theta^2 \wedge \Theta^3 \\
= \frac{b}{r^3} \Theta^2 \wedge \Theta^3
\]

\[
\Rightarrow R_{\phi \theta \phi} = R_{\theta \phi \theta} = \frac{b}{r^3} \\
\Rightarrow R_{\phi \phi \theta} = R_{\theta \phi \theta} = -\frac{b}{r^3}
\]
Riemann Tensor Components

\[
R^t_{trt} = R^r_{trr} = -R^r_{ttr} = -R^r_{trt} = \left(1 - \frac{b}{r}\right) \left[-\Phi'' - (\Phi')^2\right] - \frac{\Phi'(b'r - b)}{2r^2}
\]

\[
R^r_{\theta\theta} = R^\theta_{r\theta r} = -R^r_{\theta\theta r} = -R^\theta_{rr\theta} = \frac{b'r - b}{2r^3}
\]

\[
R^t_{\phi\phi t} = R^\phi_{t\phi t} = -R^t_{\phi t\phi} = -R^\phi_{tt\phi} = \frac{\Phi' \left(1 - \frac{b}{r}\right)}{r}
\]

\[
R^\theta_{\phi\theta\phi} = R^\phi_{\theta\phi\theta} = -R^\theta_{\phi\theta\phi} = -R^\phi_{\theta\theta\phi} = \frac{b}{r^3}
\]
Riemann Tensor Properties

\[ R_{tr\theta\phi} = -R_{rt\theta\phi} = -R_{tr\phi\theta} = R_{\theta\phi tr} \]

- Antisymmetric in \((t, r)\)
- Antisymmetric in \((\theta, \Phi)\)
- Symmetric in \((t, r)\) and \((\theta, \Phi)\)
- Only 24 independent components
- Generally in 4D has 256 components
- Governs difference in acceleration of two freely falling particles near each other
\[ G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} \]

**Ricci Curvature Tensor**

\[
R_{\alpha\beta} = R_{\alpha\beta}^{i} + R_{\alpha\beta}^{r} + R_{\alpha\beta}^{\theta} + R_{\alpha\beta}^{\phi}
\]

\[
= \left( 1 - \frac{b}{r} \right) \left[ \Phi'' + (\Phi')^2 \right] + \frac{\Phi'(b'r - b)}{2r^2} + \frac{2\Phi' \left( 1 - \frac{b}{r} \right)}{r}
\]

\[
R_{rr} = R_{rr}^{i} + R_{rr}^{r} + R_{rr}^{\theta} + R_{rr}^{\phi}
\]

\[
= \left( 1 - \frac{b}{r} \right) \left[ -\Phi'' - (\Phi')^2 \right] - \frac{\Phi'(b'r - b)}{2r^2} + \frac{b'r - b}{r^3}
\]
Ricci Curvature Tensor

\[ G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} \]

\[ R_{\theta\theta} = R^t_{\theta\theta} + R^r_{\theta r\theta} + R^\theta_{\theta r\theta} + R^\phi_{\theta\phi\theta} \]

\[ \Phi' \left( 1 - \frac{b}{r} \right) = - \frac{b' r - b}{r} + \frac{b}{2r^3} + \frac{b}{r^3} = R_{\phi\phi} \]

\[ R_{\phi\phi} = R^t_{\phi\phi} + R^r_{\phi r\phi} + R^\theta_{\phi\theta\phi} + R^\phi_{\phi\phi\phi} \]

\[ \Phi' \left( 1 - \frac{b}{r} \right) = - \frac{b' r - b}{r} + \frac{b}{2r^3} + \frac{b}{r^3} = R_{\theta\theta} \]
\[ G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} \]

**Curvature Scalar**

\[ R = -R_{tt} + R_{rr} + R_{\theta\theta} + R_{\phi\phi} \]

\[ = 2 \left( 1 - \frac{b}{r} \right) \left[ -\Phi'' - (\Phi')^2 \right] - \frac{\Phi'(b'r - b)}{r^2} - \frac{4\Phi' \left( 1 - \frac{b}{r} \right)}{r} + \frac{2(b'r - b)}{r^3} + \frac{2b}{r^3} \]
\[ G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} \]

**Einstein Tensor**

- **Einstein Tensor**  
  \[ G_{\alpha\beta} = G_{tt}, G_{rr}, G_{\theta\theta}, G_{\phi\phi} \]

- **“Ricci” Curvature Tensor**  
  \[ R_{\alpha\beta} = R_{tt}, R_{rr}, R_{\theta\theta}, R_{\phi\phi} \]

- **Curvature Scalar**  
  \[ R = -R_{tt} + R_{rr} + R_{\theta\theta} + R_{\phi\phi} \]

- **Metric**  
  \[ g_{\alpha\beta} = g_{tt}, g_{rr}, g_{\theta\theta}, g_{\phi\phi} \]
Einstein Tensor – Components

\[ G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} \]

\[ G_{tt} = R_{tt} - \frac{1}{2} R g_{tt} = \frac{b'}{r^2} \]

\[ G_{rr} = R_{rr} - \frac{1}{2} R g_{rr} = -\frac{b}{r^3} + \frac{2\Phi'}{r} \left(1 - \frac{b}{r}\right) \]

\[ G_{\theta\theta} = R_{\theta\theta} - \frac{1}{2} R g_{\theta\theta} = \left(1 - \frac{b}{r}\right) \left[ \Phi'' - \frac{b'r-b}{2r(r-b)} \Phi' + \left(\Phi'\right)^2 + \frac{\Phi'}{r} - \frac{b'r-b}{2r^2(r-b)} \right] = G_{\phi\phi} \]

\[ G_{\phi\phi} = R_{\phi\phi} - \frac{1}{2} R g_{\phi\phi} = \left(1 - \frac{b}{r}\right) \left[ \Phi'' - \frac{b'r-b}{2r(r-b)} \Phi' + \left(\Phi'\right)^2 + \frac{\Phi'}{r} - \frac{b'r-b}{2r^2(r-b)} \right] = G_{\theta\theta} \]
$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$

Stress-Energy Tensor

$$T_{\alpha\beta} = \begin{bmatrix}
\rho & 0 & 0 & 0 \\
0 & \tau & 0 & 0 \\
0 & 0 & p & 0 \\
0 & 0 & 0 & p
\end{bmatrix}$$
Equations of State

Rearrange, solve....

- Energy density
  \[ \rho = \frac{b'}{8\pi r^2} \]

- Tension
  \[ \tau = \frac{1}{8\pi r^2} \left[ \frac{b}{r} - 2(r - b)\Phi' \right] \]

- Pressure (stress)
  \[ p = \frac{r}{2} \left| (\rho - \tau)\Phi' - \tau' \right| - \tau \]
Wormhole Embedding Diagram

\[ z(r) = \pm b_0 \ln \left( \frac{r}{b_0} + \sqrt{\left( \frac{r}{b_0} \right)^2 - 1} \right), \quad b_0 = 2 \]

- Static (t=constant “slice”)
- Assume \( \theta = \pi/2 \) (equatorial “slice”)
- Only \( r, \Phi \) variable

\[
\begin{align*}
    ds^2 &= \left(1 - \frac{b}{r} \right)^{-1} dr^2 + r^2 d\phi^2 \\
    ds^2 &= dz^2 + dr^2 + r^2 d\phi^2
\end{align*}
\]
Boundary Conditions - Shape

- $b_0$ – minimum radius at throat
- Vertical at throat
- Asymptotically flat

$r$-axis
Boundary Conditions – No Horizon

Horizon - “physically nonsingular surface at which $g_{tt}$ vanishes”; defined only for spacetimes containing one or more asymptotically flat regions

- e.g. Schwarzschild metric – coordinate singularity at $r=2M$

$$ds^2 = -(1 - \frac{2M}{r}) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

- Morris-Thorne metric

$$ds^2 = -e^{2\Phi} dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

$\Phi < \infty \Rightarrow e^{2\Phi} \neq 0$
Other Boundary Conditions

- Crossing time on order of 1 year
- Acceleration and tidal acceleration on order of 1G
Geodesics

- “Geodesics” are “extremal proper time worldlines”; equations of motion that determine geodesics comprise the “geodesic equation” ~ Hartle
- Timelike – Particle freefall paths; $ds^2 < 0$
- Null – Light freefall paths; $ds^2 = 0$
**Variational Principle**

\[ ds^2 = -e^{2 \Phi} dt^2 + \frac{1}{1 - \frac{b}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

**Lagrangian**

\[ L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma}} = \left\{ e^{2\Phi} \left( \frac{dt}{d\sigma} \right)^2 - \left( 1 - \frac{b}{r} \right)^{-1} \left( \frac{dr}{d\sigma} \right)^2 - \left( 1 - \frac{b}{r} \right) \left( \frac{d\theta}{d\sigma} \right)^2 + \sin^2 \theta \left( \frac{d\phi}{d\sigma} \right)^2 \right\}^{\frac{1}{2}} \]

**Euler–Lagrange Equation**

\[ \tau_{AB} = \int_0^1 L d\sigma \]

\[ \Rightarrow -\frac{d}{d\sigma} \frac{\partial L}{\partial \left( \frac{dx^\alpha}{d\sigma} \right)} + \frac{\partial L}{\partial x^a} = 0 \Rightarrow \text{geodesics} \]
Geodesic Equation

\[
\frac{d^2 x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau}; \quad \Gamma^\alpha_{\beta\gamma} \equiv \frac{1}{2} g^{\alpha\sigma} \left( g_{\sigma\beta,\gamma} + g_{\sigma\gamma,\beta} - g_{\beta\gamma,\sigma} \right)
\]

\[\alpha, \beta, \gamma, \sigma = t, r, \theta, \phi\]

\[
\alpha = t \Rightarrow \frac{d}{d\tau} \left( e^{2\phi} \frac{dt}{d\tau} \right) = 0
\]

\[
\alpha = r \Rightarrow \frac{d}{d\tau} \left[ \left( 1 - \frac{b}{r} \right)^{-1} \frac{dr}{d\tau} \right] - \left[ \frac{d}{dr} \left( 1 - \frac{b}{r} \right)^{-1} \left( \frac{dr}{d\tau} \right)^2 + r \left( \frac{d\theta}{d\tau} \right)^2 + \sin^2 \theta \left( \frac{d\phi}{d\tau} \right)^2 \right] - \Phi' e^{2\phi} \left( \frac{dt}{d\tau} \right)^2 = 0
\]

\[
\alpha = \theta \Rightarrow \frac{d}{d\tau} \left( r^2 \frac{d\theta}{d\tau} \right) - r^2 \sin \theta \cos \theta \left( \frac{d\phi}{d\tau} \right)^2 = 0
\]

\[
\alpha = \phi \Rightarrow \frac{d}{d\tau} \left( r^2 \sin^2 \theta \frac{d\phi}{d\tau} \right) = 0
\]
Christoffel Symbols

- Describe curvature in non-Euclidean space
  - Metric is like first derivative of warp
  - Christoffel symbol is like second derivative of warp
  - Also called “connection coefficients”

- Non-zero Christoffel symbols are components of a 3-tensor

\[
\begin{align*}
\Gamma^t_{rt} &= \Gamma^t_{tr} = \frac{d\Phi}{dr} \\
\Gamma^\phi_{r\phi} &= \Gamma^\phi_{\phi r} = \frac{1}{r} \\
\Gamma^\phi_{\theta\phi} &= \Gamma^\phi_{\phi \theta} = \frac{\cos \theta}{\sin \theta} \\
\Gamma^r_{t t} &= \left(1 - \frac{b}{r}\right) \Phi' e^{2\Phi} \\
\Gamma^r_{rr} &= \frac{b'r - b}{2r(r - b)} \\
\Gamma^\theta_{r \theta} &= \Gamma^\theta_{\theta r} = \frac{1}{r} \\
\Gamma^\phi_{\phi \phi} &= -(r - b) \sin^2 \theta \\
\Gamma^r_{\theta \theta} &= b - r
\end{align*}
\]
Morris-Thorne Examples

- Zero tidal forces
- Exotic matter limited to throat
  - “Absurdly benign” wormhole
Zero Tidal Force Solution

\[ \Phi = 0 \]

\[ b = b(r) = \sqrt{b_o r} = \sqrt{r} \text{ for } b_o = 1 \]

Equations of State

\[ \rho = \frac{b'}{8\pi r^2} = 16\pi r^{\frac{5}{2}} \]

\[ \tau = \frac{1}{8\pi r^2} \left[ \frac{b}{r} - 2(r - b)\Phi' \right] = 8\pi r^{\frac{5}{2}} \]

\[ p = \frac{r}{2} |(\rho - \tau)\Phi' - \tau'| - \tau = 2\pi r^{\frac{5}{2}} \]
Zero Tidal Force Solution

- Wormhole material extends from throat to proper radial distance +/- $\infty$

- Density, tension and pressure vanish asymptotically

- Material is everywhere exotic
  - *i.e.*, $\tau > \rho > 0$ everywhere
  - Violates energy conditions
  - Need quantum field theory

“Catenoid”
Exotic Matter Limited to Throat

\[ b = b_0 \left[ 1 - \frac{(r - b_0)}{a_0} \right]^2 \]

\[ \Phi = 0 \text{ for } b_0 \leq r \leq b_0 + a_0 \]

\[ b = \Phi = 0 \text{ for } r \geq b_0 + a_0 \]

- Spacetime flat at \( r > b_0 + a_0 \)
- Tidal forces bearable
- Travel time reasonable
- BUT throat radius must be large to have meaningful wormhole
Backward Time Travel

- Time machine – Any object or system that permits one to travel to the past
- Not proven possible or impossible
- Traveler moves through wormhole at sub-light speed
- Appears to have exceeded light speed to stationary observers
- Causality violations and paradoxes (consistency and bootstrap)

From H.G. Wells, *The Time Machine*
Summary

- Theoretically reasonable
- Morris-Thorne model
  - No horizons
  - Exotic matter
  - Energy condition violations
  - Causality violation
- Much work has been done and continues to be done in this area; models abound!
Possible Questions for Future

☐ Is necessary topological change even permitted? Exotic matter required?

☐ If so, is it allowed on the quantum level?

☐ If so, can we enlarge to classical size?

☐ Morris and Thorne: “...pulling a wormhole out of the quantum foam...”
THANK YOU!

UNCW Class of 2005 (Mathematics)
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