From Here to Eternity and Back: Are *Traversable* Wormholes Possible?

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with advising from
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Phys. 495 Spring 2009
April 24, 2009
Dedicated in Memory of My Dear Friend and UNCW Alumnus
Karen E. Gross (April 11, 1982~Jan. 4, 2009)

Do Physicists Care, and Why?

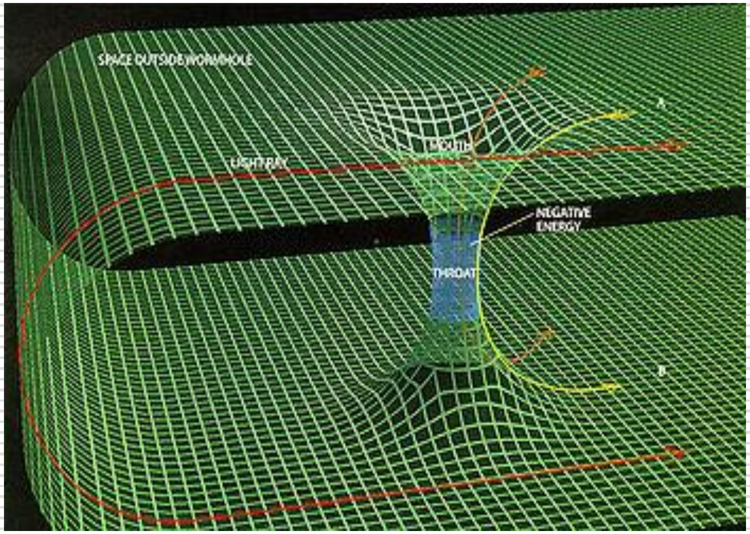
- Are wormholes possible?
  - How can we model to prove or disprove?
  - Wormholes are being taken seriously
- Possible uses?
  - Interstellar travel
  - Time travel

Outline

- History
- Models that fail
- Desirable traits
- General relativity primer
- Morris-Thorne wormhole
  - Curvature
  - Stress energy tensor
  - Boundary conditions and embeddings
  - Geodesics
  - Examples
- Time machines and future research

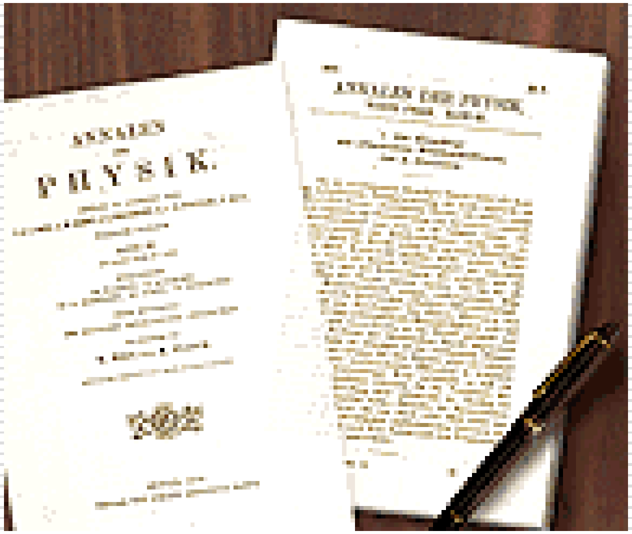
What is a Wormhole?

- "Hypothetical shortcut between distant points"
- General relativity



Historical Perspective

- Albert Einstein – general relativity ~ 1916
- Karl Schwarzschild – first exact solution to field equations ~ 1916
  - Unique spherically symmetric vacuum solution
- Ludwig Flamm ~ 1916
  - White hole solution

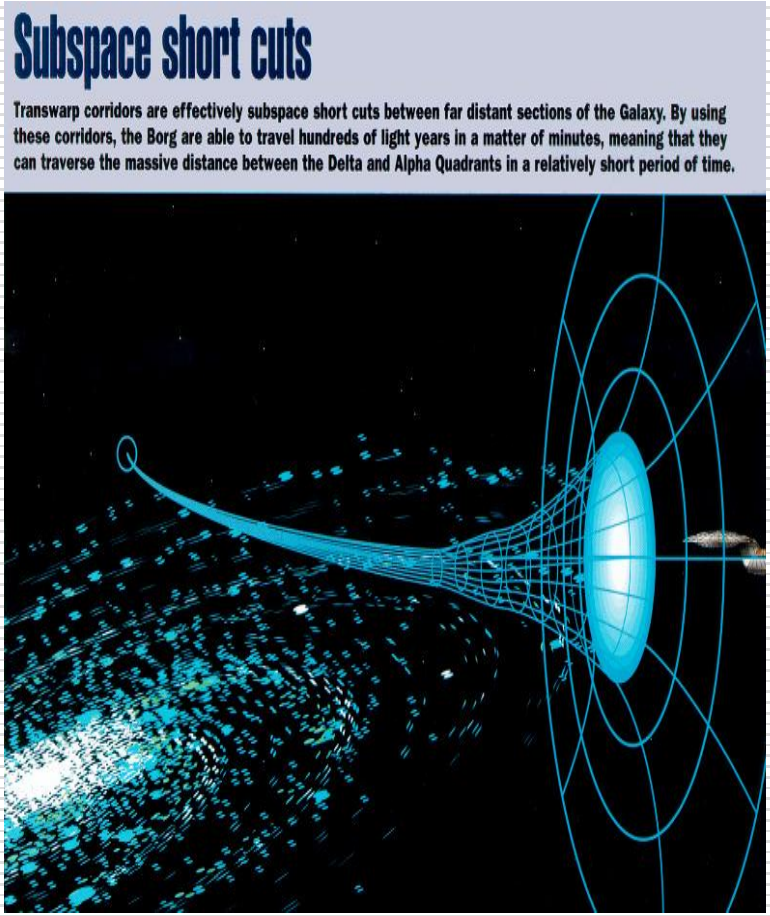


Historical Perspective

- Einstein and Nathan Rosen ~ 1935
  - “Einstein-Rosen Bridge” – first mathematical proof
- Kurt Gödel ~ 1948
  - Time tunnels possible?
- John Archibald Wheeler ~ 1950’s
  - Coined term, “wormhole”
- Michael Morris and Kip Thorne ~ 1988
  - “Most promising” – wormholes as tools to teach general relativity

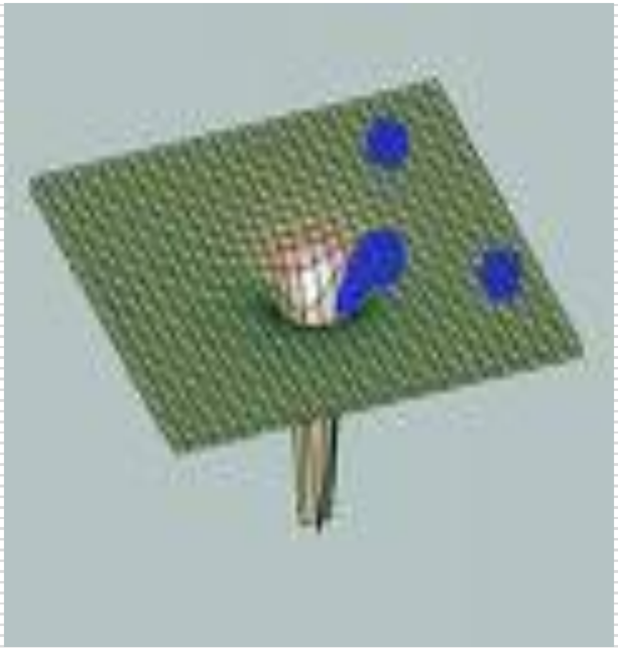
Traversable Wormholes?

- Matter travels from one mouth to other through throat
- Never observed
- BUT proven valid solution to field equations of general relativity



Black Holes Not the Answer

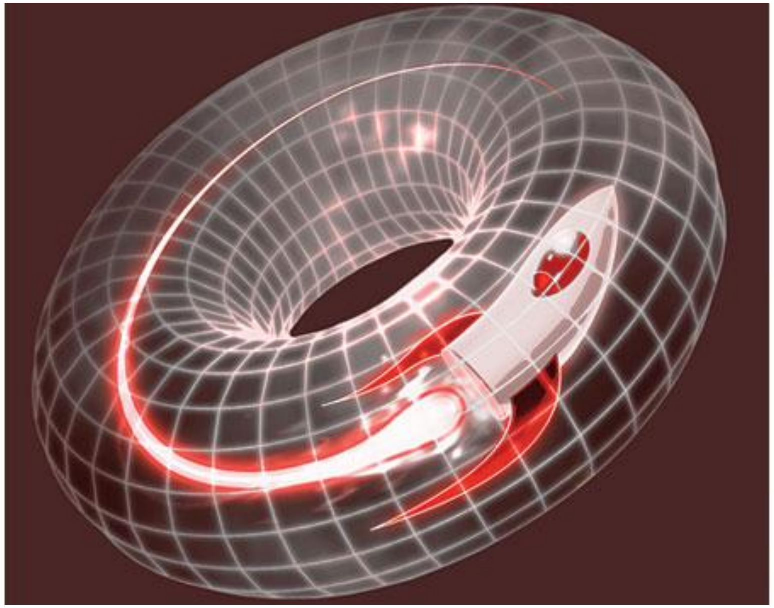
- Tidal forces too strong
- Horizons
  - One-way membranes
  - Time slows to stop
- "Schwarzschild wormholes"
  - Fail for same reasons



Simple Models – Geometric Traits

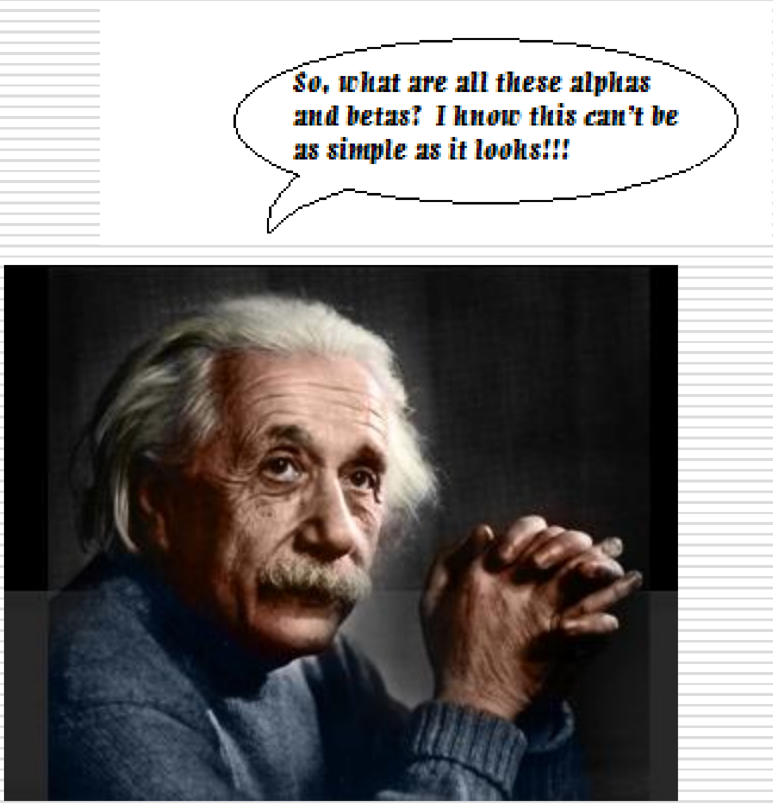
- Everywhere obeys Einstein’s field equations
- Spherically symmetric, static metric
- “Throat” connecting asymptotically flat spacetime regions
- No event horizon
  - Two-way travel
  - Finite crossing time

Simple Models – Physical Traits



- Small tidal forces
- Reasonable crossing time
- Reasonable stress-energy tensor
- Stable
- Assembly possible

A Little General Relativity Primer



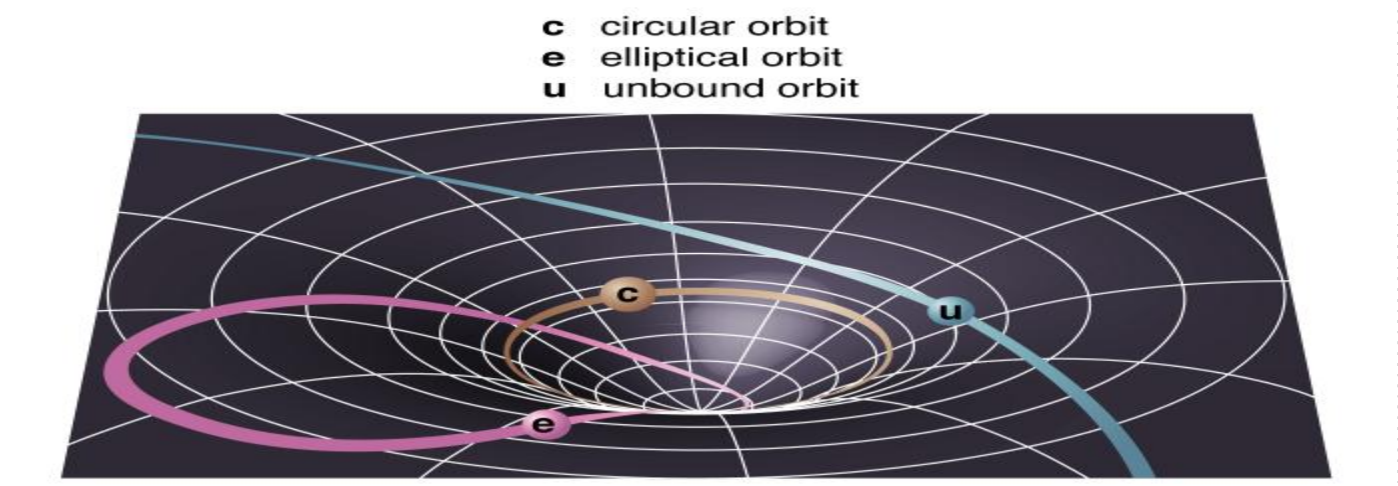
$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$$

- Field equations – relate spacetime curvature to matter and energy distribution
- Left side – Curvature
- Right side – Stress energy tensor
- Summation indices

More on General Relativity

"Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve." ~ Misner, Thorne, Wheeler, *Gravitation*



Spacetimes

"A spacetime is a four-dimensional manifold equipped with a Lorentzian metric…[of signature]…(-,+,+,+). A spacetime is often referred to as having (3+1) dimensions." ~Matt Visser, *Lorentzian Wormholes*.

Representing Spacetimes: Flat

"Line Element"

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

- Uses differentials
- Einstein summation
- Metric
- Minkowski space:

$$ds^2 = g_{tt} dt^2 + g_{xx} dx^2 + g_{yy} dy^2 + g_{zz} dz^2$$

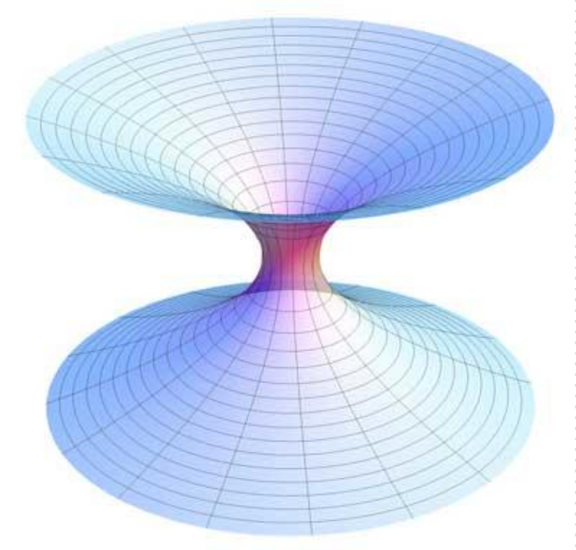
$$g_{tt} = -1; \quad g_{xx} = 1; \quad g_{yy} = 1; \quad g_{zz} = 1 \quad (cartesian)$$

$$g_{tt} = -1; \quad g_{rr} = 1; \quad g_{\theta\theta} = r^2; \quad g_{\phi\phi} = r^2 \sin^2\theta \quad (spherical)$$

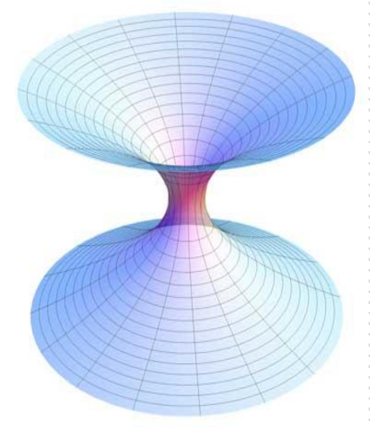
Morris-Thorne Wormhole (1988)

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{\left(1-\frac{b}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- $\Phi(r)$ – redshift function
  - Change in frequency of electromagnetic radiation in gravitational field
- b(r) – shape function



$+\infty$



$-\infty$

Properties of the Metric

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{\left(1-\frac{b}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- Spherically symmetric and static
- Radial coordinate *r* such that circumference of circle centered around throat given by $2\pi r$
- *r* decreases from $+\infty$ to $b=b_0$ (minimum radius) at throat, then increases from b_0 to $+\infty$
- At throat exists coordinate singularity where *r* component diverges
- Proper radial distance *l(r)* runs from $-\infty$ to $+\infty$ and vice versa

Morris-Thorne Metric

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$g_{\alpha\beta} = \begin{bmatrix} g_{tt} & 0 & 0 & 0 \\ 0 & g_{rr} & 0 & 0 \\ 0 & 0 & g_{\theta\theta} & 0 \\ 0 & 0 & 0 & g_{\phi\phi} \end{bmatrix} = \begin{bmatrix} -e^{2\Phi} & 0 & 0 & 0 \\ 0 & \left(1 - \frac{b}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{bmatrix}$$

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$$

Determining Curvature (Left Side)

- Do we have the desired shape?
- Cartan’s Structure Equations

“<u>Cartan I</u>”

$$d\Theta^i = -\omega^i_j \wedge \Theta^j$$

“<u>Cartan II</u>”

$$\Omega^i_j = d\omega^i_j + \omega^i_k \wedge \omega^k_j$$

Elie Joseph Cartan
(1869~1951)

Cartan I – Connection One-Forms

- *"One-forms"*

$$dx, dy, dz$$

$$pdx + qdy + rdz$$

$$\omega^i_j = -\omega^j_i$$

- *Take from Morris-Thorne metric:*

,

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{\left(1-\frac{b}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\Theta^0 = -e^{\Phi}dt$$

$$\Theta^1 = \left(1-\frac{b}{r}\right)^{-\frac{1}{2}}dr$$

$$\Theta^2 = rd\theta$$

$$\Theta^3 = r\sin\theta d\phi$$

Operations with Forms

- *"Wedge product"*

$$dt \wedge dt = dr \wedge dr = d\theta \wedge d\theta = d\phi \wedge d\phi = 0$$

$$dt \wedge dr = -dr \wedge dt$$

- *"d" Operator: k-form to (k+1)-form*

$$1-\ form$$

$$a = a(t,r) \Rightarrow da = \frac{da}{dt}dt + \frac{da}{dr}dr$$

- *Combining:*

$$\omega = a(t,r)dt \Rightarrow d\omega = \frac{da}{dt}dt \wedge dt + \frac{da}{dr}dr \wedge dt = \frac{da}{dr}dr \wedge dt$$

Calculation – Connection One-Forms

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{\left(1-\frac{b}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\Theta^0 = -e^{\Phi}dt \Rightarrow d\Theta^0 = \Phi' e^{\Phi} dt \wedge dr; \quad dt = -\frac{1}{e^{\Phi}}\Theta^0$$

$$\Theta^1 = \left(1-\frac{b}{r}\right)^{-\frac{1}{2}} dr \Rightarrow d\Theta^1 = 0; \quad dr = \left(1-\frac{b}{r}\right)^{\frac{1}{2}}\Theta^1$$

$$\Theta^2 = r d\theta \Rightarrow d\Theta^2 = dr \wedge d\theta; \quad d\theta = \frac{1}{r}\Theta^2$$

$$\Theta^3 = r\sin\theta d\phi \Rightarrow d\Theta^3 = \sin\theta dr \wedge d\phi + r\cos\theta d\theta \wedge d\phi; \quad d\phi = \frac{1}{r\sin\theta}\Theta^3$$

Calculation – Connection One-Forms

$$d\Theta^i = -\omega^i_j \wedge \Theta^j$$

$$i, j = t, r, \theta, \phi$$

$$d\Theta^0 = -\omega^0_1 \wedge \Theta^1 - \omega^0_2 \wedge \Theta^2 - \omega^0_3 \wedge \Theta^3 = \Phi' e^\Phi dt \wedge dr$$

$$\Rightarrow -\omega^0_1 \wedge \Theta^1 = \Phi' e^\Phi dt \wedge dr \Rightarrow \omega^0_1 \wedge \Theta^1 = -\Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt \wedge \Theta^1$$

$$\Rightarrow \omega^0_1 = -\Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt$$

$$\Rightarrow \omega^1_0 = \Phi' e^\Phi \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} dt$$

Matrix of One-Forms

$$\omega^i_j = \begin{bmatrix} 0 & -\Phi' e^{\Phi}\left(1-\frac{b}{r}\right)^{\frac{1}{2}} dt & 0 & 0 \\ \Phi' e^{\Phi}\left(1-\frac{b}{r}\right)^{\frac{1}{2}} dt & 0 & -\left(1-\frac{b}{r}\right)^{\frac{1}{2}} d\theta & -\sin\theta\left(1-\frac{b}{r}\right)^{\frac{1}{2}} d\phi \\ 0 & \left(1-\frac{b}{r}\right)^{\frac{1}{2}} d\theta & 0 & -\cos\theta d\phi \\ 0 & \sin\theta\left(1-\frac{b}{r}\right)^{\frac{1}{2}} d\phi & \cos\theta d\phi & 0 \end{bmatrix}$$

Calculation – Curvature Two-Forms

$$\Omega^i_{\ j} = d\omega^i_{\ j} + \omega^i_{\ k} \wedge \omega^k_{\ j}$$

$$= -\frac{1}{2} R^i_{\ mnj} \Theta^m \wedge \Theta^n$$

$$i, j, k, m, n = t, r, \theta, \phi$$

- Computed using matrix of one-forms
- Non-zero components of Riemann tensor
- Useful for computing geodesics

Calculation – Curvature Two-Forms

$$\omega_3^2 = -\cos\theta d\phi$$

$$d\omega_3^2 = \sin\theta d\theta \wedge d\phi$$

$$\omega_0^2 = \omega_3^0 = 0$$

$$\omega_1^2 = \left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\theta$$

$$\omega_3^1 = -\sin\theta\left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\phi$$

$$d\theta = \frac{1}{r}\Theta^2$$

$$d\phi = \frac{1}{r\sin\theta}\Theta^3$$

$$\Omega_3^2 = d\omega_3^2 + \omega_0^2 \wedge \omega_3^0 + \omega_1^2 \wedge \omega_3^1$$

$$= \frac{b}{r}\sin\theta d\theta \wedge d\phi$$

$$= \left(\frac{b}{r}\sin\theta\right)\left(\frac{1}{r}\right)\left(\frac{1}{r\sin\theta}\right)\Theta^2 \wedge \Theta^3$$

$$= \frac{b}{r^3}\Theta^2 \wedge \Theta^3$$

$$\Rightarrow R^\theta_{\phi\theta\phi} = R^\phi_{\theta\phi\theta} = \frac{b}{r^3}$$

$$\Rightarrow R^\theta_{\phi\phi\theta} = R^\phi_{\theta\theta\phi} = -\frac{b}{r^3}$$

Riemann Tensor Components

$$R^{t}_{rtr} = R^{r}_{ttr} = -R^{r}_{ttr} = -R^{r}_{trt} = \left(1-\frac{b}{r}\right)\left[-\Phi'' - \left(\Phi'\right)^{2}\right] - \frac{\Phi'\left(b'r-b\right)}{2r^{2}}$$

$$R^{r}_{\phi r\phi} = R^{\phi}_{r\phi r} = -R^{r}_{\phi\phi r} = -R^{\phi}_{rr\phi} = \frac{b'r-b}{2r^{3}}$$

$$R^{r}_{\theta r\theta} = R^{\theta}_{r\theta r} = -R^{r}_{\theta\theta r} = -R^{\theta}_{rr\theta} = \frac{b'r-b}{2r^{3}}$$

$$R^{t}_{\phi\phi t} = R^{\phi}_{t\phi t} = -R^{t}_{\phi t\phi} = -R^{\phi}_{tt\phi} = \frac{\Phi'\left(1-\frac{b}{r}\right)}{r}$$

$$R^{t}_{\theta\theta t} = R^{\theta}_{t\theta t} = -R^{t}_{\theta t\theta} = -R^{\theta}_{tt\theta} = \frac{\Phi'\left(1-\frac{b}{r}\right)}{r}$$

$$R^{\theta}_{\phi\theta\phi} = R^{\phi}_{\theta\phi\theta} = -R^{\theta}_{\phi\phi\theta} = -R^{\phi}_{\theta\theta\phi} = \frac{b}{r^{3}}$$

Riemann Tensor Properties

$$R_{tr\theta\phi} = -R_{rt\theta\phi} = -R_{tr\phi\theta} = R_{\theta\phi tr}$$

- Antisymmetric in (t, r)
- Antisymmetric in (θ,Φ)
- Symmetric in (t, r) and (θ, Φ)
- Only 24 independent components
- Generally in 4D has 256 components
- Governs difference in acceleration of two freely falling particles near each other

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$$

Ricci Curvature Tensor

$$R_{tt} = R^{t}_{ttt} + R^{r}_{trt} + R^{\theta}_{t\theta t} + R^{\phi}_{t\phi t}$$

$$= \left(1 - \frac{b}{r}\right)\left[\Phi'' + (\Phi')^2\right] + \frac{\Phi'(b'r - b)}{2r^2} + \frac{2\Phi'\left(1 - \frac{b}{r}\right)}{r}$$

$$R_{rr} = R^{t}_{rtr} + R^{r}_{rrr} + R^{\theta}_{r\theta r} + R^{\phi}_{r\phi r}$$

$$= \left(1 - \frac{b}{r}\right)\left[-\Phi'' - (\Phi')^2\right] - \frac{\Phi'(b'r - b)}{2r^2} + \frac{b'r - b}{r^3}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$$

Ricci Curvature Tensor

$$R_{\theta\theta} = R^{t}_{\theta t\theta} + R^{r}_{\theta r\theta} + R^{\theta}_{\theta\theta\theta} + R^{\phi}_{\theta\phi\theta}$$

$$= -\frac{\Phi'\left(1 - \frac{b}{r}\right)}{r} + \frac{b'r - b}{2r^{3}} + \frac{b}{r^{3}} = R_{\phi\phi}$$

$$R_{\phi\phi} = R^{t}_{\phi t\phi} + R^{r}_{\phi r\phi} + R^{\theta}_{\phi\theta\phi} + R^{\phi}_{\phi\phi\phi}$$

$$= -\frac{\Phi'\left(1 - \frac{b}{r}\right)}{r} + \frac{b'r - b}{2r^{3}} + \frac{b}{r^{3}} = R_{\theta\theta}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$$

Curvature Scalar

$$R = -R_{tt} + R_{rr} + R_{\theta\theta} + R_{\phi\phi}$$

$$= 2\left(1 - \frac{b}{r}\right)\left[-\Phi'' - \left(\Phi'\right)^2\right] - \frac{\Phi'\left(b'r - b\right)}{r^2} - \frac{4\Phi'\left(1 - \frac{b}{r}\right)}{r} + \frac{2\left(b'r - b\right)}{r^3} + \frac{2b}{r^3}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$$

Einstein Tensor

- Einstein Tensor $G_{\alpha\beta} = G_{tt}, G_{rr}, G_{\theta\theta}, G_{\phi\phi}$
- "Ricci" Curvature Tensor $R_{\alpha\beta} = R_{tt}, R_{rr}, R_{\theta\theta}, R_{\phi\phi}$
- Curvature Scalar $R = -R_{tt} + R_{rr} + R_{\theta\theta} + R_{\phi\phi}$
- Metric $g_{\alpha\beta} = g_{tt}, g_{rr}, g_{\theta\theta}, g_{\phi\phi}$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$$

Einstein Tensor – Components

$$G_{tt} = R_{tt} - \frac{1}{2} R g_{tt} = \frac{b'}{r^2}$$

$$G_{rr} = R_{rr} - \frac{1}{2} R g_{rr} = -\frac{b}{r^3} + \frac{2\Phi'}{r}\left(1 - \frac{b}{r}\right)$$

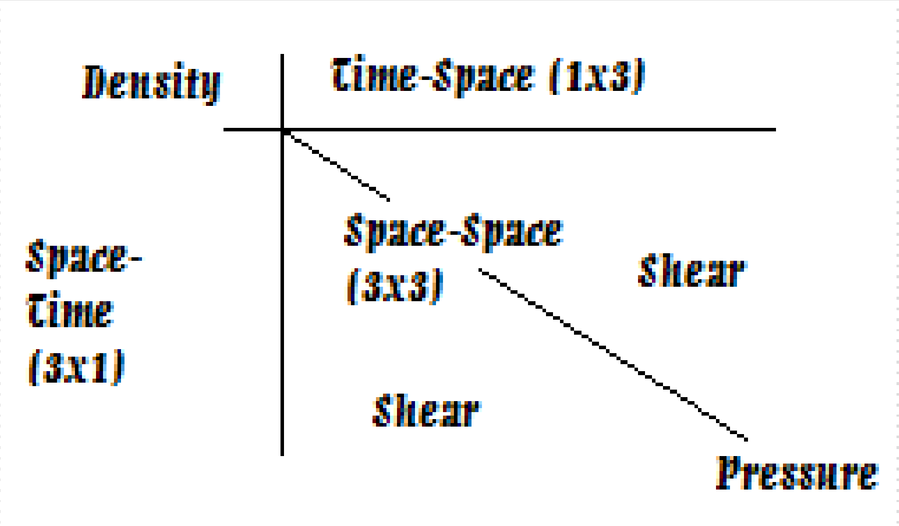
$$G_{\theta\theta} = R_{\theta\theta} - \frac{1}{2} R g_{\theta\theta} = \left(1 - \frac{b}{r}\right)\left[\Phi'' - \frac{b'r - b}{2r(r-b)}\Phi' + \left(\Phi'\right)^2 + \frac{\Phi'}{r} - \frac{b'r - b}{2r^2(r-b)}\right] = G_{\phi\phi}$$

$$G_{\phi\phi} = R_{\phi\phi} - \frac{1}{2} R g_{\phi\phi} = \left(1 - \frac{b}{r}\right)\left[\Phi'' - \frac{b'r - b}{2r(r-b)}\Phi' + \left(\Phi'\right)^2 + \frac{\Phi'}{r} - \frac{b'r - b}{2r^2(r-b)}\right] = G_{\theta\theta}$$

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$$

Stress-Energy Tensor

$$T_{\alpha\beta} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & \tau & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$



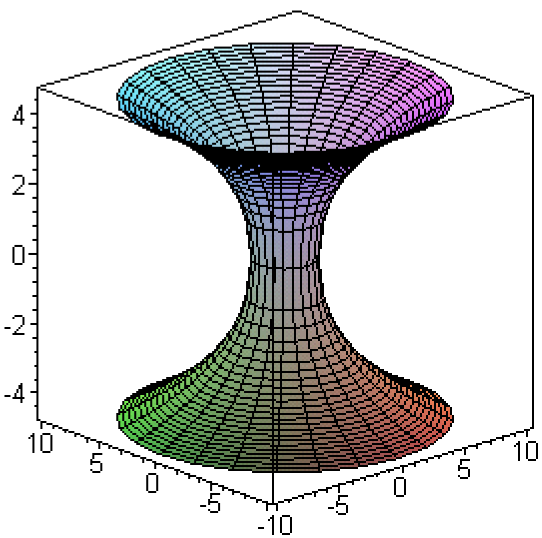
Equations of State

Rearrange, solve….

- Energy density $$\rho = \frac{b'}{8\pi r^2}$$
- Tension $$\tau = \frac{1}{8\pi r^2}\left[\frac{b}{r} - 2(r-b)\Phi'\right]$$
- Pressure (stress) $$p = \frac{r}{2}\left|(\rho - \tau)\Phi' - \tau'\right| - \tau$$

Wormhole Embedding Diagram

$$z(r) = \pm b_0 \ln\left(\frac{r}{b_0} + \sqrt{\left(\frac{r}{b_0}\right)^2 - 1}\right), b_0 = 2$$



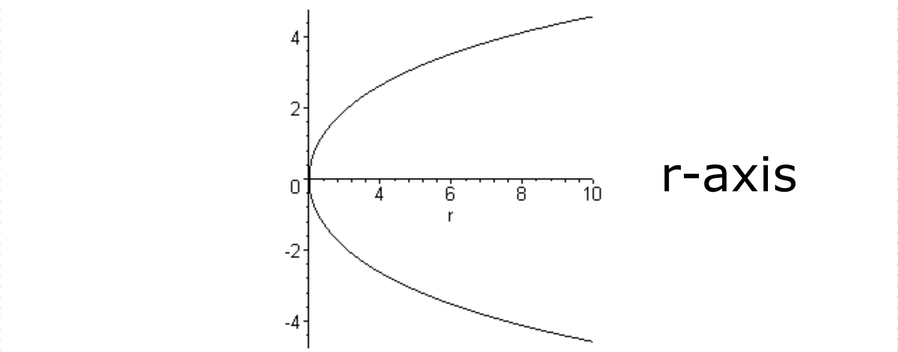
- Static (t=constant "slice")
- Assume $\theta=\pi/2$ (equatorial "slice")
- Only r,Φ variable

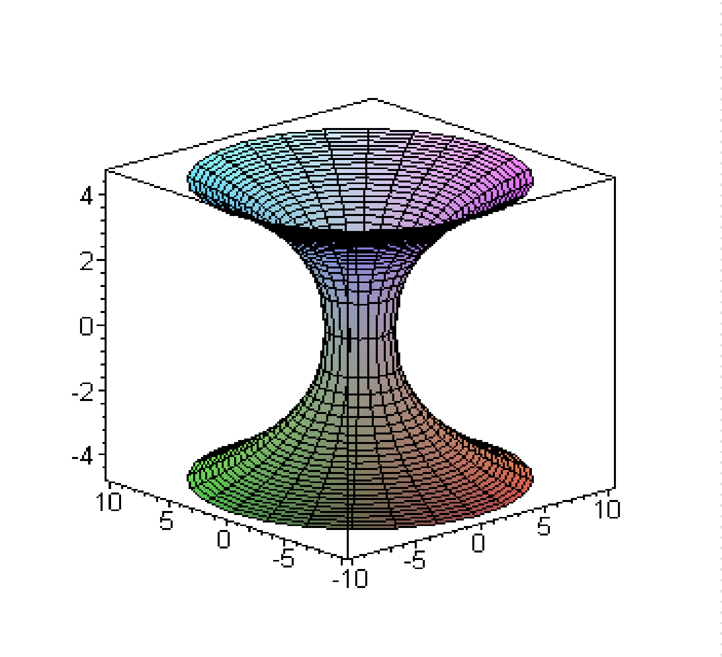
$$ds^2 = \left(1 - \frac{b}{r}\right)^{-1} dr^2 + r^2 d\phi^2$$

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2$$

Boundary Conditions - Shape

- b_0 – minimum radius at throat
- Vertical at throat
- Asymptotically flat





Boundary Conditions – No Horizon

Horizon - "*physically* nonsingular surface at which g_{tt} vanishes"; defined only for spacetimes containing one or more asymptotically flat regions

- e.g. Schwarzschild metric – coordinate singularity at *r=2M*

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Morris-Thorne metric

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

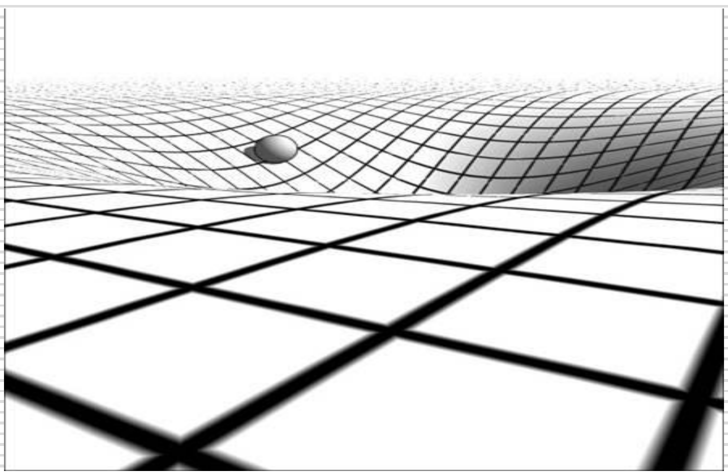
$$\Phi < \infty \Rightarrow e^{2\Phi} \neq 0$$

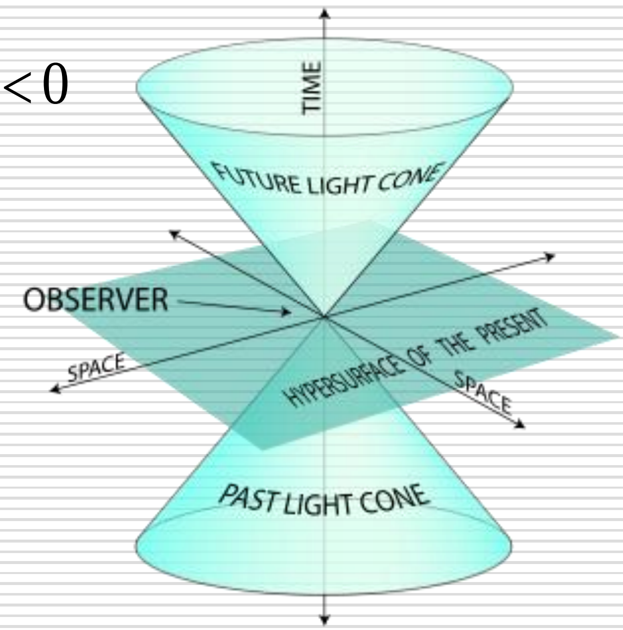
Other Boundary Conditions

- Crossing time on order of 1 year
- Acceleration and tidal acceleration on order of 1G

Geodesics

- “Geodesics” are “extremal proper time worldlines”; equations of motion that determine geodesics comprise the “geodesic equation” ~ Hartle
- Timelike – Particle freefall paths; $ds^2 < 0$
- Null – Light freefall paths; $ds^2 = 0$





LOCALLY

Variational Principle

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{\left(1-\frac{b}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- Lagrangian

$$L = \sqrt{-g_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}} = \left\{e^{2\Phi}\left(\frac{dt}{d\sigma}\right)^2 - \left(1-\frac{b}{r}\right)^{-1}\left(\frac{dr}{d\sigma}\right)^2 - r^2\left[\left(\frac{d\theta}{d\sigma}\right)^2 + \sin^2\theta\left(\frac{d\phi}{d\sigma}\right)^2\right]\right\}^{\frac{1}{2}}$$

- Euler-Lagrange Equation

$$\tau_{AB} = \int_0^1 Ld\sigma \qquad \Rightarrow -\frac{d}{d\sigma}\frac{\partial L}{\partial\left(\frac{dx^\alpha}{d\sigma}\right)} + \frac{\partial L}{\partial x^\alpha} = 0 \Rightarrow geodesics$$

Geodesic Equation

$$\frac{d^2x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau};\qquad \Gamma^\alpha_{\beta\gamma} \equiv \frac{1}{2}g^{\alpha\sigma}\left(g_{\sigma\beta,\gamma} + g_{\sigma\gamma,\beta} - g_{\beta\gamma,\sigma}\right)$$

$$\alpha,\beta,\gamma,\sigma = t,r,\theta,\phi$$

$$\alpha = t \Rightarrow \frac{d}{d\tau}\left(e^{2\Phi}\frac{dt}{d\tau}\right) = 0$$

$$\alpha = r \Rightarrow \frac{d}{d\tau}\left[\left(1-\frac{b}{r}\right)^{-1}\frac{dr}{d\tau}\right] - \left[\frac{d}{dr}\left(1-\frac{b}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 + r\left[\left(\frac{d\theta}{d\tau}\right)^2 + \sin^2\theta\left(\frac{d\phi}{d\tau}\right)^2\right] - \Phi' e^{2\Phi}\left(\frac{dt}{d\tau}\right)^2\right] = 0$$

$$\alpha = \theta \Rightarrow \frac{d}{d\tau}\left(r^2\frac{d\theta}{d\tau}\right) - r^2\sin\theta\cos\theta\left(\frac{d\phi}{d\tau}\right)^2 = 0$$

$$\alpha = \phi \Rightarrow \frac{d}{d\tau}\left(r^2\sin^2\theta\frac{d\phi}{d\tau}\right) = 0$$

Christoffel Symbols

- Describe curvature in non-Euclidean space
  - Metric is like first derivative of warp
  - Christoffel symbol is like second derivative of warp
  - Also called “connection coefficients”

- Non-zero Christoffel symbols are components of a 3-tensor

$$\Gamma^t_{rt} = \Gamma^t_{tr} = \frac{d\Phi}{dr} \qquad \Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{r} \qquad \Gamma^r_{\phi\phi} = -(r-b)\sin^2\theta$$

$$\Gamma^\phi_{r\phi} = \Gamma^\phi_{\phi r} = \frac{1}{r} \qquad \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta \qquad \Gamma^r_{\theta\theta} = b - r$$

$$\Gamma^\phi_{\theta\phi} = \Gamma^\phi_{\phi\theta} = \frac{\cos\theta}{\sin\theta} \qquad \Gamma^r_{tt} = \left(1 - \frac{b}{r}\right)\Phi' e^{2\Phi} \qquad \Gamma^r_{rr} = \frac{b'r - b}{2r(r-b)}$$

Morris-Thorne Examples

- Zero tidal forces
- Exotic matter limited to throat
  - “Absurdly benign” wormhole

Zero Tidal Force Solution

$$\Phi = 0$$

$$b = b(r) = \sqrt{b_o r} = \sqrt{r} \;\; for\; b_0 = 1$$

<u>Equations of State</u>

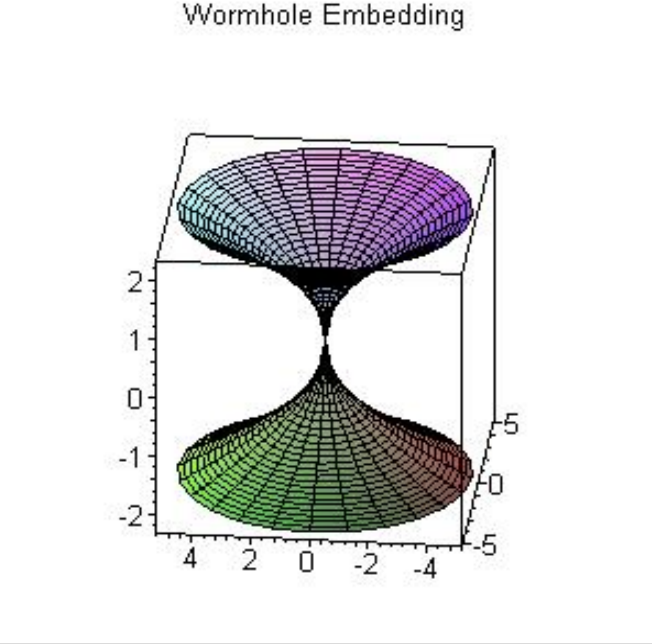
$$\rho = \frac{b'}{8\pi r^2} = 16\pi r^{-\frac{5}{2}}$$

$$\tau = \frac{1}{8\pi r^2}\left[\frac{b}{r} - 2(r-b)\Phi'\right] = 8\pi r^{-\frac{5}{2}}$$

$$p = \frac{r}{2}\left|(\rho - \tau)\Phi' - \tau'\right| - \tau = 2\pi r^{-\frac{5}{2}}$$

Zero Tidal Force Solution

- Wormhole material extends from throat to proper radial distance +/- ∞
- Density, tension and pressure vanish asymptotically
- Material is everywhere exotic
  - *i.e.*, $\tau > \rho > 0$ everywhere
  - Violates energy conditions
  - Need quantum field theory



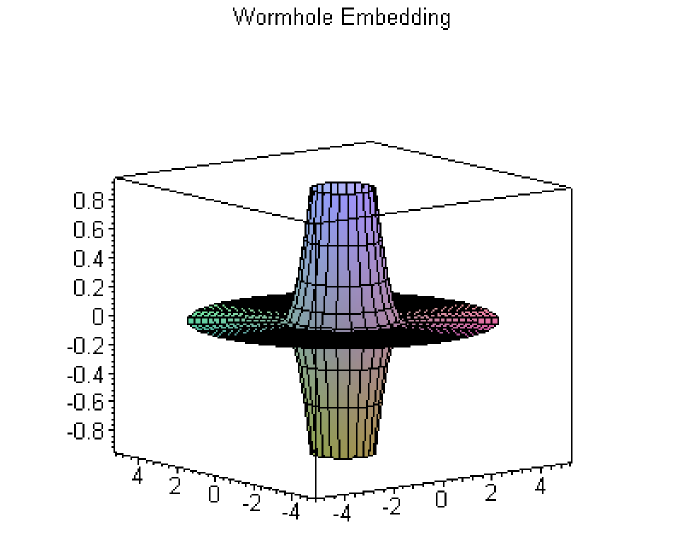
"Catenoid"

Exotic Matter Limited to Throat

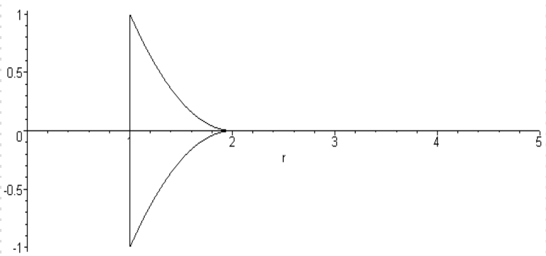
$$b = b_0 \left[1 - \frac{(r - b_0)}{a_0} \right]^2 ; \Phi = 0 \; for \; b_0 \le r \le b_0 + a_0$$

$$b = \Phi = 0 \; for \; r \ge b_0 + a_0$$

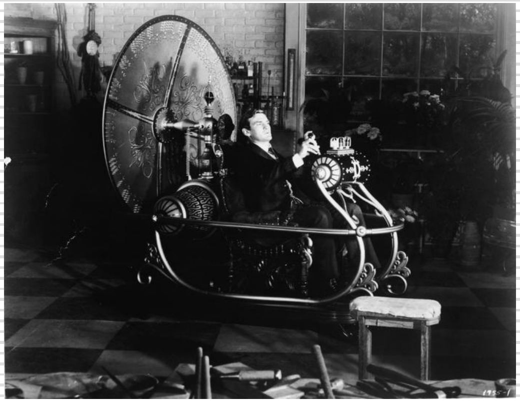
- Spacetime flat at $r > b_0 + a_0$
- Tidal forces bearable
- Travel time reasonable
- BUT throat radius must be large to have meaningful wormhole



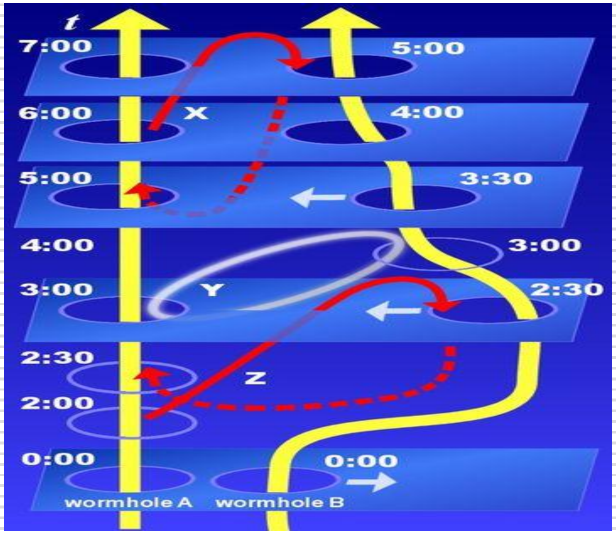
Absurdly Benign Wormhole



Backward Time Travel



From H.G. Wells,
The Time Machine



- Time machine – Any object or system that permits one to travel to the past
- Not proven possible or impossible
- Traveler moves through wormhole at sub-light speed
- Appears to have exceeded light speed to stationary observers
- Causality violations and paradoxes (consistency and bootstrap)

Summary

- Theoretically reasonable
- Morris-Thorne model
  - No horizons
  - Exotic matter
  - Energy condition violations
  - Causality violation
- Much work has been done and continues to be done in this area; models abound!

Possible Questions for Future

- Is necessary topological change even permitted? Exotic matter required?
- If so, is it allowed on the quantum level?
- If so, can we enlarge to classical size?
- Morris and Thorne: "…pulling a wormhole out of the quantum foam…"

THANK YOU!

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