Collapse and Revival in the Jaynes-Cummings-Paul Model

Departmental Honors Defense in Physics

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The Jaynes-Cummings-Paul Model

- First described in 1963 (Jaynes & Cummings).
  - Independently described in 1963 by Harry Paul.

- Second major paper in 1965 (Cummings).

- Experimentally confirmed in the 1980s.

- One method for bringing purely quantum effects into optics.
The Jaynes-Cummings-Paul Model

- Defining characteristics of the model.
  - Atom in a lossless cavity.
  - Single-mode electric field.
  - Two accessible atomic levels.
  - Atom oscillates between energy levels.
The Jaynes-Cummings-Paul Model

- Interesting properties of the model.
  - Non-zero transition probability in the absence of electric field.
  - Periodic collapse and revival of atomic oscillations.
Outline of the Project

• Construct the Jaynes-Cummings Hamiltonian.
  • Quantize the electric field.
  • Write down the atomic energy levels.
  • Work out the interaction term.
• Apply the Hamiltonian to a pair of demonstrations.
  • Definite photon states.
  • Coherent field states.
The Jaynes-Cummings Hamiltonian

\[ \hat{H}_{JC} = \hbar \omega \hat{a}^\dagger \hat{a} + \frac{1}{2} \hbar \omega_0 \hat{\sigma}_z + \hbar \lambda (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger) \]
The Jaynes-Cummings Hamiltonian

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- Put forward by Jaynes and Cummings in a pair of papers.

- A series of approximations allow for the simple form of the Hamiltonian.

- 3 components:
  - energy of the field.
  - energy of the atomic transitions.
  - energy from interaction of the field with the atom.
The Jaynes-Cummings Hamiltonian

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The Free Field Hamiltonian

\[ \hat{H}_{\text{field}} = \hbar \omega \hat{a} \hat{a}^\dagger \]
Free Field Hamiltonian

- One-dimensional cavity, boundary at $z = 0$ and $z = L$.

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2} = 0$$

- Separate variables and solve.

$$E_x(z, t) = Z(z) \cdot q(t)$$

$$= E_0 q(t) \sin(kz)$$
Free Field Hamiltonian

• We can obtain the magnetic field from Ampere’s Law.

\[ \nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \]

\[ B_y = -\frac{1}{c^2} \int \frac{\partial E_x}{\partial t} \, dz \]

\[ = \frac{\mu_0 \varepsilon_0}{k} E_0 \dot{q}(t) \cos(kz) \]
Free Field Hamiltonian

- Introduce creation and annihilation operators.

\[ \hat{a} = \frac{1}{\sqrt{2\hbar\omega}} (\omega \hat{q} + i\hat{p}) \]
\[ \hat{a}^\dagger = \frac{1}{\sqrt{2\hbar\omega}} (\omega \hat{q} - i\hat{p}) \]

- Classical functions go to quantum operators.

\[ q(t) \rightarrow \hat{q} \]
\[ \dot{q}(t) \rightarrow \hat{p} \]
We now have a quantum expression for the fields.

\[
\hat{E}_x(z, t) = E_0 \left[ \hat{a}(t) + \hat{a}^\dagger(t) \right] \sin(kz)
\]

\[
\hat{B}_y(z, t) = B_0 \left[ \hat{a}(t) - \hat{a}^\dagger(t) \right] \cos(kz)
\]

The Hamiltonian of the field may then be calculated.

\[
\hat{H} = \frac{1}{2} \int_{\text{cavity}} \left[ \varepsilon_0 \hat{E}_x^2 + \frac{1}{\mu_0} \hat{B}_y^2 \right] \, dV
\]

\[
= \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)
\]

\[
\approx \hbar \omega \hat{a}^\dagger \hat{a}
\]
The Atomic Hamiltonian

\[ \hat{H}_{\text{atom}} = \frac{1}{2} \hbar \omega_0 \hat{\sigma}_z \]
The Atomic Hamiltonian

- Limited to two accessible states implies a 2D basis.

\[ |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

- Hamiltonian is a sum over accessible energies.

\[
\hat{H}_{\text{atom}} = E_+ |+\rangle \langle +| + E_- |\rangle \langle \rangle 
\]

\[
= \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix}
\]
The Atomic Hamiltonian

- Simplify the Hamiltonian.

\[ \hat{H}_{\text{atom}} = \frac{1}{2} \left( \begin{array}{cc} E_+ + E_- & 0 \\ 0 & E_+ + E_- \end{array} \right) + \frac{1}{2} \left( \begin{array}{cc} E_+ - E_- & 0 \\ 0 & E_- - E_+ \end{array} \right) \]

\[ \hat{H}_{\text{atom}} = \frac{1}{2} (E_+ + E_-) \hat{I} + \frac{1}{2} \Delta E \hat{\sigma}_z \]

- The Hamiltonian is then expressed in terms of a Pauli matrix.

\[ \Delta E = E_+ - E_- \equiv \hbar \omega_0 \]

\[ \hat{H}_{\text{atom}} \approx \frac{1}{2} \hbar \omega_0 \hat{\sigma}_z \]
The Pauli Spin Matrices

- Pauli spin matrices.

\[ \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \]

\[ \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

- Raising & lowering operators.

\[ \hat{\sigma}_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{\sigma}_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \]
The Interaction Hamiltonian

$$\hat{H}_{\text{int}} = \hbar \lambda \left( \hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger \right)$$
The Interaction Hamiltonian

- Begin with minimal coupling:

\[ H = \frac{1}{2m} \left[ p - qA(r, t) \right]^2 + q\Phi(r, t) \]

- Apply coupling to each particle (no scalar potential):

\[ H_p = \frac{1}{2m_p} \left[ p_p^2 - e p_p \cdot A(r_p, t) - eA(r_p, t) \cdot p_p + e^2 A^2(r_p, t) \right] \]
\[ H_e = \frac{1}{2m_e} \left[ p_e^2 + e p_e \cdot A(r_e, t) + eA(r_e, t) \cdot p_e + e^2 A^2(r_e, t) \right] \]

\[ H_{\text{int}} = H_p + H_e - \frac{e^2}{4\pi\varepsilon_0} \frac{r_e - r_p}{|r_e - r_p|^3} \]
The Interaction Hamiltonian

- Introduce center of mass coordinates.

\[ R = \frac{m_p r_p + m_e r_e}{M}, \quad r = r_e - r_p \]

\[ r_p = R - \frac{\mu}{m_p} r, \quad r_e = R + \frac{\mu}{m_e} r \]

- Also, center of mass momenta.

\[ P = p_p + p_e \quad \quad p = \frac{\mu}{m_e} p_e - \frac{\mu}{m_p} p_p \]
The Dipole Approximation

\[ \mathbf{r}_p = \mathbf{R} - \frac{\mu}{m_p} \mathbf{r}, \quad \mathbf{r}_e = \mathbf{R} + \frac{\mu}{m_e} \mathbf{r} \]

• We can now make the dipole approximation to simplify the Hamiltonian.

\[ \mathbf{A}(\mathbf{r}_p, t) \sim \mathbf{A}(\mathbf{R} + \delta \mathbf{r}, t) \sim \mathbf{A}(\mathbf{R}, t) \]

• Then write out the full interaction Hamiltonian.

\[ H_{\text{int}} = \left( \frac{\mathbf{P}^2}{2M} - \frac{e^2}{4\pi\varepsilon_0} \frac{\mathbf{r}}{|\mathbf{r}|^3} \right) + \frac{1}{2\mu} [\mathbf{p} + e\mathbf{A}(\mathbf{R}, t)]^2 \]
A second formulation takes the form of a dipole in a field.

\[ H_{\text{int}} = \left( \frac{\mathbf{P}^2}{2M} - \frac{e^2}{4\pi \varepsilon_0} \frac{\mathbf{r}}{|\mathbf{r}|^3} \right) + \frac{\mathbf{p}^2}{2\mu} - \mathbf{d} \cdot \mathbf{E}(\mathbf{R}, t) \]

We can compare the two Hamiltonians through Lagrangians.

\[ H^{(0)} = \frac{1}{2\mu} [\mathbf{p} + e \mathbf{A}(\mathbf{R}, t)]^2 \]

\[ \mathcal{L}^{(0)} = \mathbf{\dot{r}} \cdot \mathbf{p} - H^{(0)} \]

\[ \mathbf{\dot{r}} = \frac{\partial H^{(0)}}{\partial \mathbf{p}} = \frac{1}{\mu} \left[ \mathbf{p} + e \mathbf{A}(\mathbf{R}, t) \right] \]
The Dipole Approximation

\[ \dot{r} = \frac{1}{\mu} \left[ p + eA(R, t) \right] \Leftrightarrow p = \mu \dot{r} - eA(R, t) \]

- The Lagrangian for minimal coupling.

\[ \mathcal{L}^{(0)} = \frac{\mu}{2} \dot{r}^2 - e\dot{r} \cdot A(R, t) \]

- Subtracting a complete time-derivative will not change variation, leading to the same equations of motion.

\[ \mathcal{L}' = \mathcal{L}^{(0)} - \frac{d}{dt} \left( -er \cdot A(R, t) \right) \]
The Dipole Approximation

- The Lagrangian for minimal coupling.

\[
\frac{d}{dt} \left[ -e\mathbf{r} \cdot \mathbf{A}(\mathbf{R}, t) \right] = -e\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{R}, t) - e\mathbf{r} \cdot \frac{d}{dt} \mathbf{A}(\mathbf{R}, t) \\
= -e\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{R}, t) - e\mathbf{r} \cdot \frac{\partial}{\partial t} \mathbf{A}(\mathbf{R}, t)
\]

- We get exactly the form we were looking for - a dipole will give exactly the same dynamics.

\[
\mathcal{L}' = \frac{\mu}{2} \dot{\mathbf{r}}^2 - e\mathbf{r} \cdot \mathbf{E}(\mathbf{R}, t)
\]
The Dipole Approximation

- Make the inverse Legendre transformation.

\[ H' = \frac{p^2}{2\mu} - er \cdot E(R, t) \]

- Now we may quantize the field & dipole.

\[ \hat{H}_{\text{int}} = -\hat{d} \cdot \hat{E} \]

\[ = -\hat{d} \cdot E_0 \left( \hat{a} + \hat{a}^\dagger \right) \sin(kz) \]

\[ = d g \left( \hat{a} + \hat{a}^\dagger \right) \]
The Dipole Operator

- Fix the dipole operator in the basis.

\[ \langle + | \hat{d} | + \rangle = \langle - | \hat{d} | - \rangle = 0 \]

\[ \langle + | \hat{d} | - \rangle = \left( \langle - | \hat{d} | + \rangle \right)^* = d \]

- The dipole operator is responsible for “moving” the atom between energy levels.
The Pauli Spin Matrices

• Raising & lowering operators.

$$\hat{\sigma}_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{\sigma}_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\hat{\sigma}_+ |+\rangle = 0, \quad \hat{\sigma}_- |-\rangle = 0$$

$$\hat{\sigma}_+ |-\rangle = |+\rangle, \quad \hat{\sigma}_- |+\rangle = |-\rangle$$

$$\hat{d} = d (\hat{\sigma}_+ + \hat{\sigma}_-)$$
The Rotating-Wave Approximation

- Multiply out the interaction Hamiltonian.

\[ \hat{H}_{\text{int}} = \hbar \lambda (\hat{\sigma}_+ + \hat{\sigma}_-) (\hat{a} + \hat{a}^\dagger) \]
\[ = \hbar \lambda (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a} + \hat{\sigma}_+ \hat{a}^\dagger + \hat{\sigma}_- \hat{a}^\dagger) \]

- The operators gain time-dependence in the interaction picture.

\[ \hat{a}^\dagger(t) = \hat{a}^\dagger e^{i\omega t} \quad \hat{a}(t) = \hat{a} e^{-i\omega t} \]
\[ \hat{\sigma}_+(t) = \hat{\sigma}_+ e^{i\omega_0 t} \quad \hat{\sigma}_-(t) = \hat{\sigma}_- e^{-i\omega_0 t} \]
The Interaction Picture

- States in the interaction picture evolve in time slightly differently that in the Schrödinger picture.

\[
\frac{d}{dt} | \Psi_I(t) \rangle = \frac{i}{\hbar} \hat{H}_{0,S} | \Psi_I(t) \rangle + e^{i\hat{H}_{0,S}t/\hbar} \frac{d}{dt} | \Psi_S(t) \rangle
\]

\[
= \frac{i}{\hbar} \hat{H}_{0,S} | \Psi_I(t) \rangle + e^{i\hat{H}_{0,S}t/\hbar} \left( -\frac{i}{\hbar} \hat{H}_S | \Psi_S(t) \rangle \right)
\]

\[
= e^{i\hat{H}_{0,S}t/\hbar} \hat{V}_S e^{-i\hat{H}_{0,S}t/\hbar} | \Psi_I(t) \rangle
\]

\[
= \hat{V}_I(t) | \Psi_I(t) \rangle
\]
The Interaction Picture

• The state vectors in the interaction picture evolve in time according to the interaction term only.

\[ \frac{d}{dt} |\Psi_I(t)\rangle = \hat{V}_I(t) |\Psi_I(t)\rangle \]

• It can be easily shown through differentiation that operators in the interaction picture evolve in time according only to the free Hamiltonian.

\[ \frac{d\hat{O}_I}{dt} = \frac{i}{\hbar} [\hat{H}_{0,I}, \hat{O}] + \left( \frac{\partial \hat{O}_I}{\partial t} \right) \]
The Rotating-Wave Approximation

- The interaction Hamiltonian now carries oscillating phase terms.

\[ \hat{H}_{\text{int}} = \hbar \lambda \left( \hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a} + \hat{\sigma}_+ \hat{a}^\dagger + \hat{\sigma}_- \hat{a}^\dagger \right) \]

\[ = \hbar \lambda \left( \hat{\sigma}_+ \hat{a} e^{i(\omega_0 - \omega)t} + \hat{\sigma}_+ \hat{a}^\dagger e^{i(\omega_0 + \omega)t} + \right. \]

\[ \left. \hat{\sigma}_- \hat{a} e^{-i(\omega_0 + \omega)t} + \hat{\sigma}_- \hat{a}^\dagger e^{-i(\omega_0 - \omega)t} \right) \]

- Setting the detuning \( \Delta = \omega - \omega_0 \) to 0 removes time-dependence.

\[ \hat{H}_{\text{int}} = \hbar \lambda \left( \hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger \right) \]
The Jaynes-Cummings Hamiltonian

- We now have the full Jaynes-Cummings Hamiltonian.

\[ \hat{H}_{JC} = \hbar \omega \hat{a}^\dagger \hat{a} + \frac{1}{2} \hbar \omega \hat{\sigma}_z + \hbar \lambda (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger) \]
Demonstration 1:
Definite Photon States

\[ |\Psi(t)\rangle = C_+(t) |+\rangle |n\rangle + C_-(t) |-\rangle |n+1\rangle \]
The Jaynes-Cummings Hamiltonian

- We now have the full Hamiltonian.

\[ \hat{H}_{JC} = \hbar \omega \hat{a}^{\dagger} \hat{a} + \frac{1}{2} \hbar \omega \hat{\sigma}_z + \hbar \lambda (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^{\dagger}) \]

- 2 commuting terms.

\[ \hat{H}_0 = \hbar \omega \hat{a}^{\dagger} \hat{a} + \frac{1}{2} \hbar \omega \hat{\sigma}_z, \quad \hat{H}' = \hbar \lambda (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^{\dagger}) \]

- Schrödinger equation in the interaction picture.

\[ i\hbar \frac{d}{dt} |\Psi(t)\rangle = \hat{H}' |\Psi(t)\rangle \]
Definite Photon States

- The atomic states may be in a linear combination of the two energy levels.

\[ |\Psi(t)\rangle = C_+ |+\rangle |n\rangle + C_- |-\rangle |n+1\rangle \]

- Only 2 modes of transition.

*Stimulated emission*  
\[ |+\rangle |n\rangle \rightarrow |-\rangle |n+1\rangle \]

*Stimulated absorption*  
\[ |-\rangle |n\rangle \rightarrow |+\rangle |n-1\rangle \]
Definite Photon States

- Solving the Schrödinger equation and equating coefficients yields two coupled differential equations.

\[ \dot{C}_+(t) = -i\lambda\sqrt{n+1} C_-(t) \]

\[ \dot{C}_-(t) = -i\lambda\sqrt{n+1} C_+(t) \]

- These are easily solved with initial conditions. For instance, choose \(|\Psi(0)\rangle = |+\rangle \Rightarrow C_+ = 1, C_- = 0\).

\[ C_+(t) = \cos(\sqrt{n+1}\lambda t) \]

\[ C_-(t) = -i\sin(\sqrt{n+1}\lambda t) \]
Definite Photon States

- The wave function of the total system oscillates in time.

\[ |\Psi(t)\rangle = \cos(\lambda\sqrt{n+1} t) |+\rangle |n\rangle - i \sin(\lambda\sqrt{n+1} t) |-\rangle |n+1\rangle \]

- The probability amplitudes are given through inner products and also oscillate in time.

\[
P_+(t) = |C_+|^2 = \cos^2(\lambda\sqrt{n+1} t) \]

\[
P_-(t) = |C_-|^2 = \sin^2(\lambda\sqrt{n+1} t) \]

\[ P_+(t) + P_-(t) = 1 \]
Oscillation of the probability amplitudes in time.
Definite Photon States

- We are interested in measuring the atomic population inversion $W(t)$ (the expectation value of the inversion operator).

$$W(t) = \langle \Psi(t) | \hat{\sigma}_z | \Psi(t) \rangle$$

$$= | C_+ |^2 - | C_- |^2$$

$$= \cos^2(\lambda \sqrt{n + 1} t) - \sin^2(\lambda \sqrt{n + 1} t)$$

$$W(t) = \cos (2\lambda \sqrt{n + 1} t)$$
Definite Photon States

Atomic inversion for several periods and a range of electric field strengths.
Definite Photon States

• An interesting property of the model is a non-zero transition probability in the absence of electric field.

\[ W(t) \mid_{n=0} = \cos(2\lambda t) \]

\[ |(0 \mid +) \mid \Psi(t) \rangle |^2 = P_+^{(0)}(t) = \cos^2(\lambda t) \]

\[ |(1 \mid -) \mid \Psi(t) \rangle |^2 = P_-^{(0)}(t) = \sin^2(\lambda t) \]
Demonstration 2: Coherent Photon States

\[ |\Psi(t)\rangle = \sum_{n=0}^{\infty} C_n \left( C_+ |+\rangle + C_- |-\rangle \right) |n\rangle \]
Coherent States

- “Near classical” photon states.
- Superposition of photon number states.
- $|\alpha|^2 = N$ is mean photon number.

$$|\psi_{\text{field}}\rangle = e^{-N/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$P(n) = |\langle n | \psi_{\text{field}} \rangle|^2 = e^{-N} \frac{N^n}{n!}$$
Coherent States
Coherent States

• The general state of the coherent system is a direct product of the atom and field states.

\[ |\psi_{\text{field}}\rangle = \sum_{n=0}^{\infty} C_n |n\rangle \quad |\psi_{\text{atom}}\rangle = C_+ |+\rangle + C_- |-\rangle \]

\[ |\Psi(t)\rangle = |\psi_{\text{atom}}\rangle \otimes |\psi_{\text{field}}\rangle = \sum_{n=0}^{\infty} C_n [C_+ |+\rangle + C_- |-\rangle] |n\rangle \]

• We will choose a similar initial condition as before.

\[ |\Psi(0)\rangle = \sum_{n=0}^{\infty} C_n |+\rangle |n\rangle \]
Coherent States

- Solve the Schrödinger equation again with the initial condition to obtain the general wave function.

\[ |\Psi(t)\rangle = \sum_{n=0}^{\infty} C_n \left\{ \cos(\lambda \sqrt{n+1} t) |+\rangle |n\rangle - i \sin(\lambda \sqrt{n+1} t) |-\rangle |n+1\rangle \right\} \]
Coherent States

- This leads to transition probabilities that oscillate, but also consist of superpositions of photon states.

\[
P_+(t) = |\langle + | \Psi(t) \rangle|^2 = \sum_{n=0}^{\infty} e^{-N} \frac{N^n}{n!} \cos^2 (\lambda \sqrt{n + 1} t)
\]

\[
P_-(t) = |\langle - | \Psi(t) \rangle|^2 = \sum_{n=0}^{\infty} e^{-N} \frac{N^n}{n!} \sin^2 (\lambda \sqrt{n + 1} t)
\]
Coherent States

- Collapse and revival are strongly displayed in the atomic inversion of coherent states.

\[ W(t) = P_+(t) - P_-(t) = e^{-N} \sum_{n=0}^{\infty} \frac{N^n}{n!} \cos(2\lambda \sqrt{n + 1} t) \]
Coherent States

- Collapse and revival are approx. periodic over longer time-scales.
Coherent States

• More well-defined envelopes for large mean photon number.
Experimental Confirmation

- Experimentally confirmed in the 1980s.

- Rubidium maser, 2.5 Kelvin cavity, Q factor of $6 \times 10^7$.

- Large principal quantum number allows for only 2-level transitions.

Graphs & information from PRL vol. 58, no. 4, 26 January 1987, pp. 353--356.
Experimental Confirmation

FIG. 1. Scheme of the experimental setup.

Graphs & information from PRL vol. 58, no. 4, 26 January 1987, pp. 353--356.
Conclusions & Extensions of the Jaynes-Cummings-Paul Model
Conclusions

- Collapse and revival are uniquely quantum mechanical in nature.

- Spontaneous emission is uniquely quantum mechanical.

- Simplified model allows for basic understanding about photon/atom interactions.

- The assumptions are very general and easily expounded upon.
Extensions of the Model

- Collapse and revival with nonzero detuning $\Delta$.
- Cavity damping viz. photon loss (non-infinite Q factor).
- Multi-photon transitions.
- Time-dependent coupling constant $\lambda(t)$. 
Thank You!

Thank You!


Thank You!

